Marks

[10] **1.** Give an explicit description of a Turing machine that takes as input, $x \in \{0,1\}^*$, and (1) accepts x if the first character of x equals the last character, and (2) rejects x if not. You should **explicitly write** your choice of $Q, \Sigma, \Gamma, q_0, q_{\text{accept}}, q_{\text{reject}}, \delta$ and **intuitively explain** how the machine works. For example, you should write $\Sigma = \{0,1\}$, since this is the input alphabet.

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[10] **2.** Let $\mathcal{P} = \mathcal{I} = \{1, 2, 3, \ldots\}$, the set of positive integers.

(a) Can there be a Result function with the property that every language in $\mathcal{I}$ is accepted by some element of $\mathcal{P}$? Explain.

(b) Let Result(p, i) (for $p \in \mathcal{P}$ and $i \in \mathcal{I}$) be defined to be `yes` if $p > i$, `no` if $p < i$, and `loops` if $p = i$. For each $p \in \mathcal{P}$, describe the language that p accepts. Is any p a decider? Describe a language not accepted by any $p \in \mathcal{P}$.

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[10] **3.** In class we showed that $|S| < |2^S|$ for any set S, where 2^S is the set of all subsets of S. We argued that otherwise there is a bijection $f: S \to 2^S$, and then we considered:

$$T = \{s \in S \mid s \notin f(s)\}.$$

How do we obtain a contradiction? Explain.

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[10] **4.** Any string over $\{0, 1, A\}$ is uniquely expressible as $n_1An_2A \ldots n_kAn_{k+1}$, where $n_1, \ldots, n_k$ are strings over $\{0, 1\}$.

(a) Give a high level description of a Turing machine that on input $w \in \{0, 1, A\}^*$, with $w = n_1An_2A \ldots An_{k+1}$, moves the tape head to the n_1-th occurrence of A if it exists, where we view n_1 as an integer in binary notation. Roughly how many extra tape symbols will you need? Show that you can perform this task in time order $|w|^2$.

(b) Explain the relevance of an algorithm similar to Part (a) to designing a universal Turing machine, U. [Hint: U's input contains a description of all the values of δ, the transition rule, of a Turing machine to be simulated.]

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The End

Be sure that this examination has 6 pages including this cover

The University of British Columbia

Midterm Examinations - October 2011

Computer Science 421/501

Closed book examination Time: 50 minutes

Name _____ **Signature** _____

Student Number _____ **Instructor's Name** _____

Section Number _____

Special Instructions:

Calculators, notes, or other aids may not be used. Answer questions on the exam. This exam is two-sided!

Rules governing examinations

1. Each candidate should be prepared to produce his library/AMS card upon request.

2. Read and observe the following rules:

No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

CAUTION - Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

(a) Making use of any books, papers or memoranda, other than those authorized by the examiners.

(b) Speaking or communicating with other candidates.

(c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

3. Smoking is not permitted during examinations.

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2		10
3		10
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Total		40