CPSC 421/501 Nov 19, 2020

- Some more NP-complete problems
- How to solve P vs. NP (?)
  - There are 2^{2^n} Boolean functions on n-variables
  - For n large, less than $\sqrt{2^{2^n}}$ can be computed by a circuit of size $2^n/2n$
  - If 3COLOUR $\in$ P, then 3COLOUR has polynomial size circuits; can we refute the conclusion???
  - Questions about circuit/formula size/depth of Boolean functions are still wide open.

Breakout room problems:

(1) Show that

$$\text{PARTITION} = \left\{ \langle n_1, \ldots, n_m \rangle \;\middle|\; \text{for some } I \subset [m],\ \sum_{i \in I} n_i = \sum_{i \notin I} n_i \right\}$$

is NP-complete

(2) Show that

$$\text{VERTEX-EXPANSION} = \left\{ \langle G, a, b \rangle \;\middle|\; \text{there is a } A \subset V_G \text{ with } |A| = a \text{ and } |\Gamma(A)| \geq b \right\}$$

is NP-complete

(3) Show that $CLIQUE = \{ \langle G, k \rangle \mid G$ has a clique of size $k \}$ is NP-complete

(4) Show that every Boolean function on n variables can be computed by a formula of

(a) Size $\leq n 2^n$

AND

(b) Depth $\leq \lceil \log_2 n \rceil n$

Graph theory terminology:

Let $G=(V,E)$ be an (undirected) graph. Let $A \subset V$ be a subset.

(1) $\Gamma(A)$ = "neighbours of A"

$$= \{ v \in V \mid v \notin A \text{ and some edge is incident upon } v \text{ and some element of } A \}$$

(2) A is a clique if any two elements of A are joined by an edge.