- Finish the Cook-Levin theorem:
  - if SAT $\in$ P, then NP = P
  - if 3SAT $\in$ P, then NP = P

- In practice we are more interested in 3SAT

- Formalize NP-completeness and reductions

- Many languages are NP-complete:
  3SAT, 3-COLOUR, 4-COLOUR, etc.
  SUBSET-SUM, PARTITION, etc.
  EXPANSION, etc.
  VERTEX COVER, etc.
  etc.
Breakout Room Problems

① Say that SAT ∈ P. Show that given a Boolean formula, \( f = f(x_1, \ldots, x_n) \), one can find \( x_1^*, \ldots, x_n^* \in \{ T, F \} \) s.t. if \( f \in \text{SAT} \), then \( f(x_1^*, \ldots, x_n^*) = T \).

② If \( L_1 \leq_p L_2 \) by an \( O(n^3) \) reduction, and \( L_2 \leq_p L_3 \) " " \( O(n^5) \) " " , then \( L_1 \leq_p L_3 \). How much time does the reduction require?

③ Say that 3COLOUR is NP-complete. Show that 4COLOUR " " " " . Is 2COLOUR NP-complete?
4) Show that for fixed \(X_1, \ldots, X_6\),
\[X_1 \lor X_2 \lor X_3 \lor X_4 \lor X_5 \lor X_6 = \top\]
iff (for those values of \(X_1, \ldots, X_6\)) the formula
\[(X_1 \lor X_2 \lor y_1) \land (\neg y_1 \lor X_3 \lor y_2) \land (\neg y_2 \lor X_4 \lor y_3) \land (\neg y_3 \lor X_5 \lor X_6)\]
is satisfiable.

5) Say that \(L \leq \text{NP}\) and we can prove that \(L \leq \text{P} \Rightarrow \text{NP} = \text{P}\). Does this mean \(L\) is necessarily \(\text{NP-complete}\)?
(6) Any Boolean function $f(x_1, \ldots, x_n)$ can be written as

$$(\text{clause}_1) \text{ or } (\text{clause}_2) \text{ or } \ldots \text{ or } (\text{clause}_{2^n})$$

where

$$\text{clause}_i = \text{literal}_{i,1} \text{ AND } \text{literal}_{i,2} \text{ AND } \ldots \text{ AND } \text{literal}_{i,n}$$

where each literal $\text{literal}_{i,j}$ is one of

$x_1, x_2, \ldots, x_n, \neg x_1, \neg x_2, \ldots, \neg x_n$

(and $\neg$ is negation),

i.e. as a CNF of size $2^n$ (or less)

and width $n$ (or less).

(7) Any Boolean function $f(x_1, \ldots, x_n)$ can be written in DNF of size $2^n$ and width $n$. 