CPSC 421/501 Nov. 12, 2020

- Finish the Cook-Levin theorem:
  - if SAT $\in$ P, then NP = P
  - if 3SAT $\in$ P, then NP = P

- In practice we are more interested in 3SAT

- Formalize NP-completeness and reductions

- Many languages are NP-complete:

  3SAT, 3-COLOUR, 4-COLOUR, etc.

  SUBSET-SUM, PARTITION, etc.

  EXPANSION, etc.

  VERTEX COVER, etc.

  etc.

Breakout Room Problems

(1) Say that SAT $\in$ P. Show that given a Boolean formula, $f = f(x_1, \ldots, x_n)$, one can find $x_1^*, \ldots, x_n^* \in \{T, F\}$ s.t. if $f \in$ SAT, then $f(x_1^*, \ldots, x_n^*) = T$.

(2) If $L_1 \leq_p L_2$ by an $O(n^3)$ reduction,

and $L_2 \leq_p L_3$ " " $O(n^3)$ " ,

then $L_1 \leq_p L_3$. How much time does the reduction require?

(3) Say that 3COLOUR is NP-complete.

Show that 4COLOUR " " " .

Is 2COLOUR NP-complete?

(4) Show that for fixed $x_1, \ldots, x_6$,

$$x_1 \text{ OR } x_2 \text{ OR } x_3 \text{ OR } x_4 \text{ OR } x_5 \text{ OR } x_6 = T$$

iff (for those values of $x_1, \ldots, x_6$) the formula

$(x_1 \text{ OR } x_2 \text{ OR } y_1)$ AND

$(\neg y_1 \text{ OR } x_3 \text{ OR } y_2)$ AND

$(\neg y_2 \text{ OR } x_4 \text{ OR } y_3)$ AND

$(\neg y_3 \text{ OR } x_5 \text{ OR } x_6)$

is satisfiable.

(5) Say that $L \in NP$ and we can prove that $L \in P \Rightarrow NP = P$. Does this mean L is necessarily NP-complete?

(6) Any Boolean $f = f(x_1, \ldots, x_n)$ can be written as

(clause_1) OR (clause_2) OR ... OR (clause_{2^n})

where

$\text{clause}_i = \text{literal}_{i,1}$ AND $\text{literal}_{i,2}$ AND ... AND $\text{literal}_{i,n}$

where each $\text{literal}_{i,j}$ is one of

$x_1, x_2, \ldots, x_n, \neg x_1, \neg x_2, \ldots, \neg x_n$

(and $\neg$ is negation),

i.e. as a CNF of size 2^n (or less) and width n (or less).

(7) Any Boolean $f = f(x_1, \ldots, x_n)$ can be written in DNF of size 2^n and width n.