CPSC 421/501 Oct 15, 2020

§3.1 — recognize versus decide

§3.2 — k-tape Turing Machines

$$\delta : Q \times \Gamma^k \to Q \times \Gamma^k \times \{L, R, S\}^k$$

- Non-deterministic

$$\delta : Q \times \Gamma \to \text{Power}(Q \times \Gamma \times \{L, R\})$$

§3.3 - Descriptions of
- Graphs
- Boolean formulas
- etc.

Chapter 4:

§4.1 : Decidable Problems (examples)

§4.2 : Undecidable Problems

- Universal Turing Machines can recognize

A_{TM}, $HALT_{TM}$

- But A_{TM}, $HALT_{TM}$ are undecidable.

Breakout Room Problems:

① Give high-level or implementation level of Turing machine to decide:

$$\text{PRIMES} = \{2, 3, 5, 7, 11, 13, 17, \ldots\}$$

$$\text{TIMES} = \left\{ a\#b\#c \;\middle|\; \begin{array}{l} a, b, c \in \{0,1\}^*, \\ a \cdot b = c \text{ as} \\ \text{base 2 numbers} \end{array} \right\}$$

$$\text{3COLOR} = \left\{ \langle G \rangle \;\middle|\; \begin{array}{l} G \text{ is a graph} \\ \text{that can be 3-colored} \end{array} \right\}$$

② Give an algorithm (deterministic Turing machine) to recognize

$$\left\{ \langle p \rangle \;\middle|\; \begin{array}{l} p = p(x, y, z) \text{ is a polynomial over} \\ \text{the integers such that } p(a, b, c) = 0 \\ \text{for some } a, b, c \in \mathbb{N} \end{array} \right\}$$

③ What is a reasonable way to describe (over some finite alphabet):

- a Boolean formula ?
- a polynomial $p(x,y)$ of x,y with integer coefficients ?
- a DFA ?
- a Turing machine ?

④ Is the set of Turing machines countable?

⑤ Is the set of "Turing machines algorithms" (where you identify two machines that "run the same algorithm") countable?

Last time: For $\{0^n 1^n \mid n = 1, 2, \ldots\}$ we built: