

- Set Theory Subtleties

Not subtle: If T is uncountable, and there is a surjection $S \rightarrow T$, then S is uncountable.

Subtle: If there is an injection $S \rightarrow T$, then there is a surjection $T \rightarrow S$.

- Russell's Paradox

- Related Paradoxes (Section 6 of handout)

- Start Finite Automata (§ 1.1 of Textbook)

BREAKOUT ROOM PROBLEMS

① If S is countable, and there is a surjection $S \rightarrow T$, then T is countable. (Prove this)

② Is $\text{POWER}(\{a,b\}^*) \cup \mathbb{N}$ countable?

③ Is there a bijection

$$[2]^{\mathbb{N}} \rightarrow [3]^{\mathbb{N}},$$

and can you describe one?

(4) Let F be the set of functions $\mathbb{N} \rightarrow \mathbb{N}$ that can be described in English

(assuming a fixed, precise interpretation of English). Is F countable or uncountable?

(5) Let $\mathbb{N}^{\mathbb{N}}$ be the set of functions $\mathbb{N} \rightarrow \mathbb{N}$. Is $\mathbb{N}^{\mathbb{N}}$ countable or uncountable?