The University of British Columbia

Final Exam, December 5, 2017

CPSC 421

Closed book examination | Time: 2.5 hours

Last Name ____________________ **First** ______________ **Signature** ____________________

Student Number ____________________

Special Instructions:

Two two-sided 8.5 x 11 sheets of notes allowed.

1		16
2		15
3		15
4		15
5		15
Total		76

[16] **1**. Circle either T (true) or F (false).
Circle either T for true, or F for false, for each of the statements below:

The set of functions $\{0, 1\} \to \mathbb{Z}$ is countable. T F

The set of functions $\mathbb{Z} \to \{0, 1\}$ is countable. T F

"more interesting"

If $L' \leq_{\mathrm{P}} L$ then $L' \in \mathrm{P}^L$. T F

Instance of L' poly time alg → hit button for L

If L is undecidable and recognizable, then L's complement is unrecognizable. T F

If L and L' are in PSPACE then $L^* \cap L'$ is also in PSPACE. T F

If L is PSPACE-complete and $L' \in \mathrm{NP}$, then $L' \leq_P L$. T F

If $L \in \mathrm{P}$, then there are polynomial size circuits for L. T F

$L' \in NP$

$\Rightarrow L' \in PSPACE \Rightarrow L' \leq_p L$

[15] **2.** Give a Turing machine that takes as input, $x \in \{0,1\}^*$, and (1) accepts x if x begins with a 0 and x has exactly two more 0's than it has 1's, and (2) rejects x otherwise. You must **explain how your machine works**, and **explicitly write** your choice of $Q, \Sigma, \Gamma, \delta, q_0, q_{\rm accept}, q_{\rm reject}$. To describe δ, you may (1) list its values, or (2) use a diagram as used in Sipser's textbook (and class), or (3) give a table of its values as done in class and the solutions to Homework 10.

[15] **3.** Let L be the language of descriptions of a sequence of positive integers $n_1, \ldots, n_k, m, t$ such that there is an $I \subset [k] = \{1, \ldots, k\}$ and a $J \subset [k]$ for which

$$\left(\sum_{i \in I} n_i \right) \; + \; m \left(\sum_{j \in J} n_j \right) = t.$$

Show that L is NP-complete; you may use the fact that SUBSET-SUM and ~~PARTITION~~ are known to be NP-complete.

[15] **4.** Fix an integer $k \in \mathbb{N}$. Let L be the language of strings over $0, 1$ whose k-th last symbol is a 1 (and whose length is therefore at least k), i.e.,

$$L = \left\{ x1y \;\middle|\; x, y \in \{0,1\}^* \text{ and } |y| = k - 1 \right\}.$$

(a) Write an NFA that recognizes L and has at most $k + 1$ states, and explain how your NFA works.

(b) Prove that any DFA recognizing L has at least 2^k states.

[18] **5**. **Short Problems**. Each question is worth 3 points. Answer each question and **justify your anwer. No credit will be given for a simple yes or no.**

(a) Let B be any PSPACE-complete language. Is $P^B \subset$ PSPACE?

(b) Is NP contained in $\mathrm{P}^{\mathrm{3SAT}}$?

(c) Let L be the language of descriptions $\langle M, w, q \rangle$ of a (deterministic) Turing machine M, an input w to M, and a state q of the Turing machine such that q is reached during the computation of M on input w. Is L recognizable?

(d) Let SNEAKY-PSPACE be the descriptions $\langle M, w, 1^s \rangle$ where M accepts w within space 1^s. Prove that if $L \in$ PSPACE, then L has a polynomial time reduction to SNEAKY-PSPACE.

(e) If L is in P and L' is any language, then is $\{s \mid st \in L \text{ for some } t \in L'\}$ necessarily in P? [Hint: Consider $\{0^n 1^n\}$.]

The End