Today! Scivitch’s Theorem, Number of Configurations

Tic-Tac-Toe 3x3!

Play X first

Win!

sit.

for all

Some small amount of space + a lot of time

Win in game for player 1

(assume player 1, player 2 alternate)

\[ \exists \text{ a first move player 1 s.t. } \forall \text{ moves player 1 } \forall \text{ moves player 2} \]

2 player game \( \Rightarrow \) player 1 wins.

\[ \text{There exists a winning move for player 1.} \]
\[ p = \text{player 1} \]
\[ q = \text{player 2} \]

\[ \text{function} (p_1, q_1, p_2, q_2, \ldots) = \text{win player} \]

\[ \mathbb{B} \mathbf{F} = \{ \langle f \rangle \mid f \text{ is a Boolean formula s.t.} \]
\[ \forall x_1 \exists x_2 \forall x_3 \exists x_4 \ldots \forall x_{n-1} \exists x_n \]
\[ f(x_1, \ldots, x_n) = \]
\[ \text{true} \]
\[ \text{e.g.} \]
\[ f(x_1) = \]
\[ \frac{x}{x} \]
\[ \frac{x}{x} \]

\[ \text{e.g.: } f = f(x_1, \ldots, x_n) \]
\[ \text{but only depends on } x_2, x_4, x_5, x_7, \ldots \text{ even vars} \]
\[ \forall x_1 \exists x_2 \forall x_3 \exists x_4 \ldots \forall x_{n-1} \exists x_n \]
\[ f(x_1, x_2, \ldots, x_n) = \]
\[ \exists x_2 x_4 x_5 x_7 \ldots x_n \text{ s.t. } f(x_1, \ldots, x_n) = \]
\[ \exists f(x_1, x_2, x_3, \ldots, x_n) \in \text{SAT} \]
\[ \uparrow \]
\[ \hat{p}(n) \]
Claim: If \( L \) is decidable in \( 2n^5 \) Non-det Space, then \( L \) is decidable in \( O(n^{10}) \) Space.

\[ O(4^{2^m}) \]

\[ \text{PSPACE} = \text{NPSPACE} \]

i.e.

If you have a deterministic Time that runs in space \( 2n^5 \),

\[ \begin{array}{c}
\text{tape} \\
\text{board} \end{array} \]

\[ \text{Total number of configurations} = \]

\[ \leq C \cdot 2n^5 \]

Last time: step 1

step some config

same config
What is $T_m$, M, that is non-deterministic?

To accept w input, could we take longer that # configs to accept? 

This is all independent of $q_f$

So: non-det $T_m$.
Algorithm

`config`!  middle config  possible  \( \rightarrow \) step \( T \)

`config`  \( \uparrow \)

fixed

given

\( \exists \) some middle config

\( \uparrow \)

\( \frac{T}{2} \) steps

can get

go through all possible configs

\( \uparrow \)

\( \frac{T}{2} \) steps

Analogous

T steps

can get

go through all possible steps

C \( 2n^5 \)

How much space config config config

\( \log_2 (C 2n^5) = O(n^5) \)

Some 2n^5 cells

\( \bigstar \)