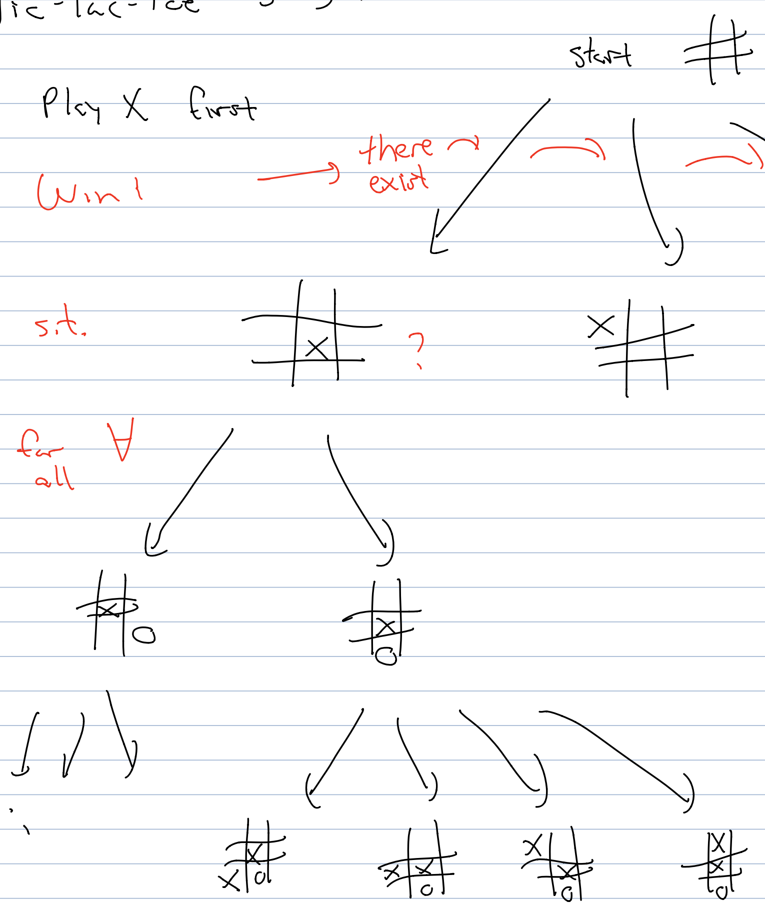
Today: Savitch's Theorem, Number of Configurations

Tic-Tac-Toe 3×3!

Play X first



Win! ⟶ there exist ⟶ ⟶ ⟶ $\exists$

s.t.

for all $\forall$

Some small amount of space

+ a lot of time

Win in game for player 1

(assume player 1, player 2 alternate)

$\Longleftrightarrow$ $\exists$ a first move player 1 s.t. $\forall$ moves of player $\exists$ move player 1 $\forall$ moves player 2

. . . . (player 1 wins)

function of the moves that is a win

2 player game $\Longleftrightarrow$ player 1 wins $\exists .. \forall \exists \forall$ . . .

p = player 1
q = player 2

$$\exists p_1 \forall q_1 \exists p_2 \forall q_2 \exists \cdots$$

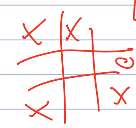
$$\boxed{\begin{array}{c}\text{function}(p_1, q_1, p_2, q_2 \ldots) \\ \Rightarrow \text{win player 1}\end{array}}$$

$$\widetilde{QBF} = \left\{ \langle f \rangle \;\middle|\; \begin{array}{l} f \text{ is a Boolean formula, s.t.} \\ \forall x_1 \exists x_2 \forall x_3 \exists x_4 \cdots \forall x_{n-1} \exists x_n \\ f(x_1, \ldots, x_n) = \text{true} \end{array} \right\}$$

$f(x_1, \ldots, x_n)$

n even

e.g.



e.g. $f = f(x_1, \ldots, x_n)$

but only depends on $x_2, x_4, x_6, x_8, \ldots$ even vars

$$\forall x_1 \exists x_2 \forall x_3 \exists x_4 \cdots \forall x_{n-1} \exists x_n \quad f(x_1, \ldots, x_n) = T$$

$$\iff \exists x_2 x_4 x_6 x_8 \cdots x_n \text{ s.t. } f(x_1, \ldots, x_n) = T$$

$$\iff f(x_2, x_4, x_6, \ldots x_n) \in SAT$$

$f(n)$

Claim: If L is decidable in $2n^5$ Non-det Space,

then L is decidable in $O(n^{10})$ Space.

$O(f^2(n))$

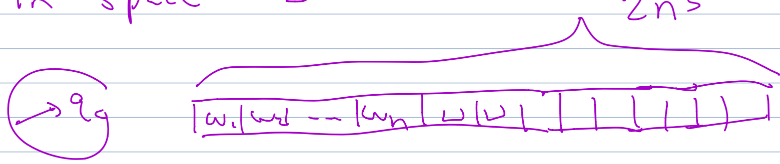
i.e.

PSPACE = NPSPACE

I.e.

If you have a deterministic T.m. that runs

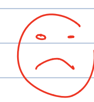
in space $2n^5$



step 1

Q = states

Γ = tape symbols

Total number of configurations = 

board  ;

\# of states, \# stuff we write on tape, \# tape head positions

$$|Q| \cdot |\Gamma|^{2n^5} \cdot 2n^5$$

$$\leq C^{2n^5}$$ some constant

Last time: step 1

⋮

step some config

⋮ same config

" " " "

step 1 ← config 1

config 2

see same config after $\left(1 + \begin{matrix}\text{\# of} \\ \text{configurations}\end{matrix}\right)$

then loop

step $1+\left(\begin{matrix}\text{\# of} \\ \text{configs}\end{matrix}\right)$

What is T.m. M, that is non-deterministic?

To accept w input, could we take longer that #configs to accept?

" " " " , do we need to " " "

step 1

possible state 1

← state

← same state

← same state

q_{acc}

don't need

q_{acc}

(This is all independent of δ)

So: non-det T.m.

step 1

step C^{2n^5}

input w

can we reach q_{acc}

Algorithm

config 1

fixed

given

middle config

possible config end

step T

go through all possible configs

all possible

$\exists$ some middle config

$T/2$ steps

can get

$T/2$ steps

if a config is in q_{acc}

for those in q_{acc}

go through all possible steps

step 1 or 0

and

step $T/4$

step $T/2$

step $3T/4$

step T

C^{2n^5}

How much space

config

config

how to remember?

1 1 2 2

$2n^5$ cells

$$\log_2\left(C^{2n^5}\right) = O(n^5)$$