**Make sure!**

\[ L_1 \leq_p L_2 \]

\[ f : \Sigma_1 \rightarrow \Sigma_2 \text{ in poly time.} \]

Last time

\[ f : \begin{cases} 
\text{strong rep} \\
3\text{CNF}
\end{cases} \rightarrow \langle \overline{G}, a, b \rangle \]

1. \( s \circ 3\text{SAT} \Rightarrow f(s) \in \text{VERTEX-EXPANSION} \)

2. \( s \nvDash 3\text{SAT} =) f(s) \notin \text{VERTEX-EXPANSION} \)

3. \( s \circ 3\text{SAT} \Rightarrow \begin{cases} 
\text{build } 2m \text{ I/F vertices} \\
\text{build } n \cdot B \text{ vertices}
\end{cases} \]

4. **3SAT** = \( \left\{ \langle g \rangle \mid \text{Boolean formula in 3CNF form that is satisfiable} \right\} \)

5. \( s \nvDash 3\text{SAT} \Rightarrow \begin{cases} 
\text{write } a = n = \# \text{ variables} \\
\text{b = see homework} \\
\text{maybe } nB + C \text{, } B = n+m+1
\end{cases} \)
Today: Start Ch 8: Space

= 

Given a TM \( M \) \( M \) runs in space \( f(n) \),

\[ f : \mathbb{N} \to \mathbb{N} \] if on any input, \( w \), \( M \) uses

or \( \mathbb{Z} \to \mathbb{Z}_n \)

at most \( O(f(|w|)) \) space

\[ \text{SPACE}(f(n)) \]

space = maximum # of cells used in

the entire algorithm (whether

or not we actually change their content).

\[ \text{PSPACE} = \text{Polynomial Space} = \bigcup_{k \in \mathbb{N}} \text{SPACE}(n^k) \]

non-deterministic

Given a TM \( M \) \( M \) runs in space \( f(n) \),

\[ f : \mathbb{N} \to \mathbb{N} \] if on any input, \( w \), \( M \) uses

or \( \mathbb{Z} \to \mathbb{Z}_n \)

at most \( O(f(|w|)) \) space in any computation path.

\[ \text{NSPACE}(f(n)) \]

\[ \text{NPSPACE} = \text{Non-deterministic Poly Space} = \bigcup_{k \in \mathbb{N}} \text{NSPACE}(n^k) \]
Fact: $\text{NPSPACE} = \text{PSPACE}$

Savitch's Thm: $\text{NSPACE}(n^k) \subseteq \text{SPACE}(n^{2k})$

Break: How complicated is $3 \times 3$ tic-tac-toe?

Is it more complicated than $\text{chess}$?

$3^{(19^2)} - 9 \cdot 19 \times 19$

How complicated is $4 \times 4 \times 4$ tic-tac-toe?

$7 \times 7 \times 7 \times 7$

How many configurations are there in $3 \times 3$ tic-tac-toe?

Upper bounds: $\leq 3^9$ squares, $X, O, \text{alternate}$

How many configurations of a $7 \times 7 \times 7$ board are there with each square either $\{X, O, \text{none}\}$?

$3^{(7^4)}$

$7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7$

?
JF-game: 20 piles of chips, 1024 chips in each pile

- 2 players, alternate in moves,
  each move is many chips in any number of piles
- player to first clear out all piles wins...

```
# confgs: \[ \frac{1}{\text{pile 1}} \frac{1}{\text{pile 2}} = \frac{1}{\text{pile 20}} \]

Game is not complex...
```

Back to TM's:

Say TM, $M$, runs in $SPACE \leq n^3$

How long could $M$ take?  have symbol $= \Gamma$

```
\[ Q \quad n^3 \]
```

```
# poss confgs = \[ \leq |Q| \times |\Gamma|^3 \times n^3 \]
```

```
\text{time 1} \quad \text{time 2}
```

```
\text{config} \quad \text{config} \quad \text{you loop!}
```
time $M$ takes <

What if $M$ is non-deterministic?