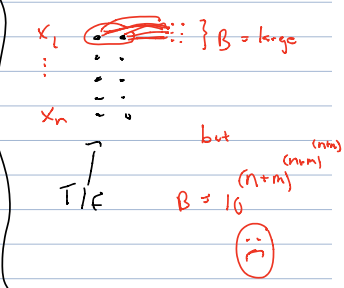
Make sure:

$L_1 \leq_p L_2$

$f: \Sigma_1^* \to \Sigma_2^*$ in poly time

3SAT



Last time

not ☹

f: (string rep 3CNF) → ⟨G, a, b⟩

(1) $s \in 3SAT \Rightarrow f(s) \in$ VERTEX-EXPANSION

and

(2) $\Leftarrow$

or

(2') $s \notin 3SAT \Rightarrow f(s) \notin$ VERTEX-EXPANSION

$g(x_1, \ldots, x_n)$ → graph G, integer a, integer b ← straightforward, poly time

① build $2n$ T/F vertices

② build $n \cdot B$ vertices

"T/F enforcement" vertices

"dummy vertices"

+ edges

③ build each of m clauses:

1 vertex

3 edges

④ write $a = n$ = # Bool vars

b = see homework

(maybe $nB + C$, $B = n+m+1$)

$3SAT = \{ \langle g \rangle \mid$ Boolean formula in 3CNF form that is satisfiable $\}$

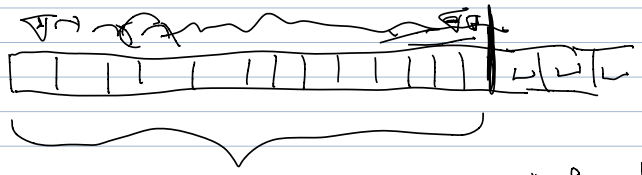
$s \notin 3SAT \Leftrightarrow$

$\{ s \mid$ either $s \neq \langle g \rangle$ for g in 3CNF or $s = \langle g \rangle$ but g is not satisfiable $\}$

Today: Start Ch 8: Space

=

Given a Tim., M, M runs in space $f(n)$,
$f: \mathbb{N} \to \mathbb{N}$ or $\mathbb{Z}_2 \to \mathbb{Z}_{>0}$ if on any input, w, M uses at most $O(f(|w|))$ space



SPACE($f(n)$)

space = maximum # of cells used in the entire algorithm (whether or not we actually change their content).

=

$$PSPACE = \text{Polynomial Space} = \bigcup_{k \in \mathbb{N}} SPACE(n^k)$$

Given a non-deterministic Tim., M, M runs in space $f(n)$,
$f: \mathbb{N} \to \mathbb{N}$ or $\mathbb{Z}_2 \to \mathbb{Z}_{>0}$ if on any input, w, M uses at most $O(f(|w|))$ space in any computation path.

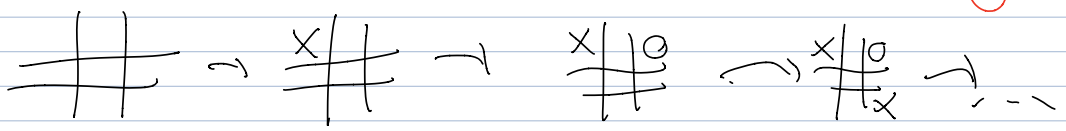
NSPACE($f(n)$)

$$NPSPACE = \text{Non-determ Polynomial Space} = \bigcup_{k \in \mathbb{N}} NSPACE(n^k)$$

Fact: NPSPACE = PSPACE

Savitch's Thm: $NSPACE(n^k) \subseteq SPACE(n^{2k})$

Break: How complicated is 3x3 tic-tac-toe?



Is it more complicated than
- chess
- go $3^{(19^2)}$ 19x19

How complicated is 4x4x4 tic-tac-toe ?

7x7x7x7 " ?

How many configurations are there in 3x3 tic-tac-toe ?

Upper bounds: $\leq 3^9$ 9 squares, X, O, ⊔, alternate

How many configurations of a # board are there with each square either {X, O, ⊔} ? 3^9

How many config ... 7x7x7x7 board ...
? $3^{(7^4)}$

... 7x7x7x7x7x7x7x7x7 ...
?

JF-game: 20 piles of chips, 1024 chips in each pile

- 2 players, alternate in moves,
  each move as many chips in any number of piles
- player to first clear out all piles wins...

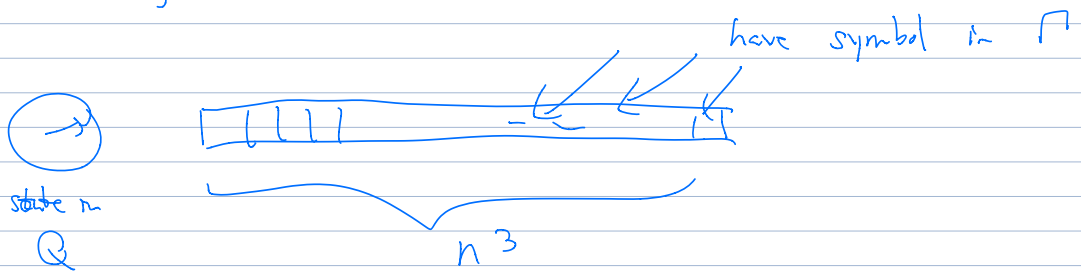
configs: $\underbrace{\quad}_{\text{pile 1}}\ \underbrace{\quad}_{\text{pile 2}}\ \cdots\ \underbrace{\quad}_{\text{piles 20}}$ 1025^{20}

Game is not complex...

Back to TM's:

Say T.m., M, runs in SPACE $\leq n^3$

How long could M take?



poss configs $=$ ☹

" " " $\leq |Q| \times |\Gamma|^{n^3} \times n^3$

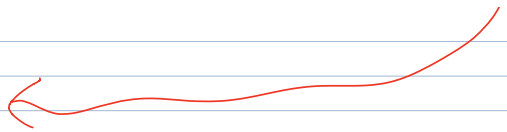
states · tape · tape head

time 1
time 2

← config
← same config as

← you loop!

time M takes $\leq$



What if M is non-det ??