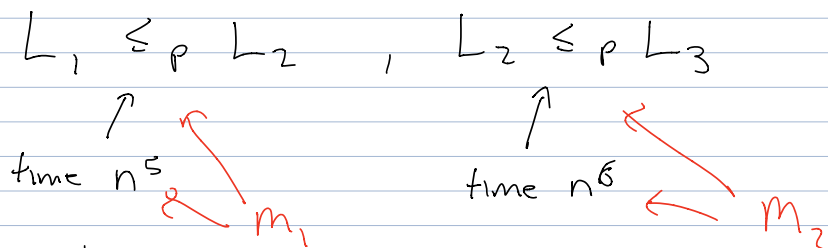Midterm: — Rubric for Q4 changed as of 11:50 am today

— For regrades, please print out your solution(s) and talk to me or TAs and indicate error in marking by Thursday, Nov 21

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— Finish Ch 7, start Ch 8:

One more NP-completeness proof...

Subtleties:

$$L_1 \leq_p L_2 \quad , \quad L_2 \leq_p L_3$$

time $n^5$     $M_1$       time $n^6$     $M_2$

then combine

do $M_1$, then $M_2$

get   $L_1 \leq_p L_3$.   How much time could this take?

In worst case — —

$L_1$ input $w$,

output of $M_1$ could be string as long as $|w|^5$

input $M_2$   "   "   "   "   "   "   $|w|^5$

output of "   "   "   "   "   "   "   $(|w|^5)^6 = |w|^{30}$.

One more NP-completeness:

Last time took Boolean formula ⎫
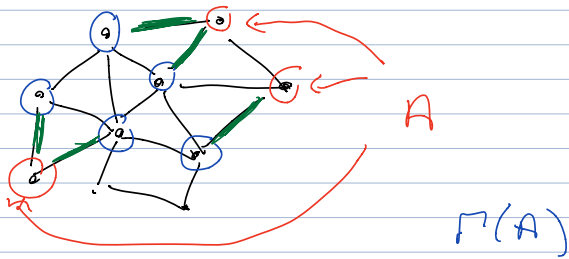$$f = f(x_1, \ldots, x_n)$$
in 3CNF ⎬ ⟶ problem in SUBSET-SUM

e.g. $(x_1 \text{ or } x_2 \text{ or } x_3) \text{ and } (x_1 \text{ or } \neg x_2 \text{ or } \neg x_3)$

⟶ 10101101 -- )
1001 -- )
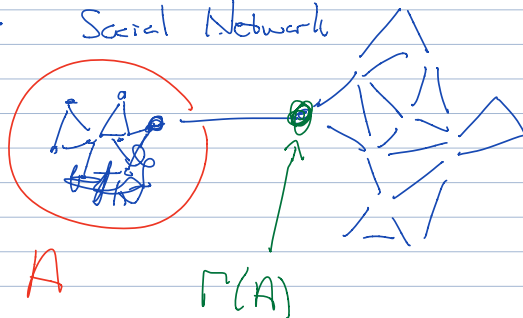⋮ )
target 7111 77777

_____

Graph theoretic problem:

Vertex Expansion: $G = (V, E)$,

given $A \subset V$, $\Gamma(A) = $ set of neighbours of A

$$= \left\{ v \in V \;\middle|\; \begin{array}{l} v \notin A, \text{ but } v \\ \text{is adjacent to} \\ \text{some element of } A \end{array} \right\}$$



A

$\Gamma(A)$

e.g.   Social Network



A      $\Gamma(A)$                    clustering

VERTEX-EXPANSION

$$= \left\{ \langle G, a, b \rangle \middle| \begin{array}{l} G \text{ is graph} \\ \text{Is there a subset, } A, \text{ of} \\ \text{vertices size } a, \text{ s.t.} \\ \Gamma(A) \geq b \end{array} \right\}$$

for 3SAT
reduction
$a = n = $ # vars

Claim: VERTEX-EXPANSION is NP-complete.

(1) V-E is in NP: just "guess" a subset of vertices
of size $a$.

If $n$-vertices, $a = n/2$, # of such subsets $\binom{n}{n/2} \sim 2^n / \sqrt{n} \cdot \text{const}$

(too big to be a
poly time alg)

Given $f = f(x_1, \ldots, x_n)$ in 3CNF

$\rightsquigarrow$ vertex expansion question

s.t. $f$ is Satisfiable iff vertex exp question
is in VERTEX-EXPANSION

e.g.

$(X_1 \text{ or } X_2 \text{ or } X_3)$ AND
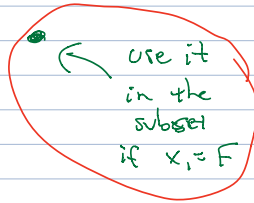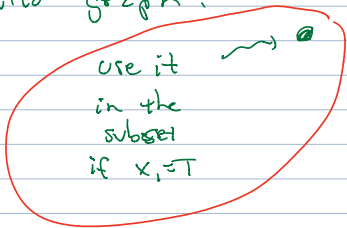
$(X_1 \text{ or } \neg X_2 \text{ or } \neg X_3) \ldots$

Create in

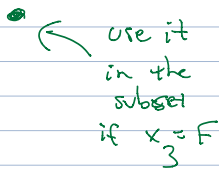① "gadget" that expresses
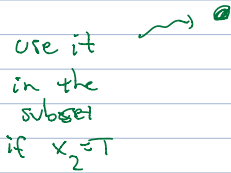$X_1 \leftrightarrow T/F, X_2 \leftrightarrow T/F, \ldots$

② "gadget" that checks whether
or not all clauses are true

in
SUBSET-SUM

gadget $\longleftrightarrow$ one of the
digits

Build graph:

use it
in the
subset
if $x_1 = T$

use it
in the
subset
if $x_1 = F$

enforce
T or F

$\gtrless$ ?

enforce
each clause
is satisfied

use it
in the
subset
if $x_2 = T$

use it
in the
subset
if $x_2 = F$

use it
in the
subset
if $x_3 = T$

use it
in the
subset
if $x_3 = F$

---

Build graph:

use it
in the
subset
if $x_1 = T$

use it
in the
subset
if $x_1 = F$

enforce
T or F

enforce
each clause
is satisfied

0
0
8
etc.

use it
in the
subset
if $x_2 = T$

use it
in the
subset
if $x_2 = F$
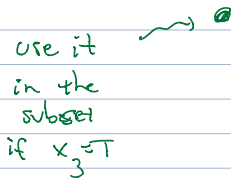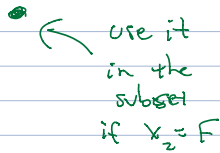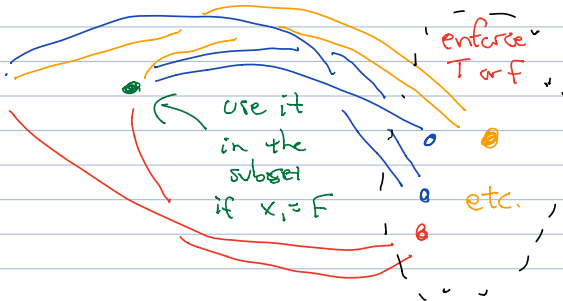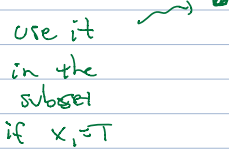
use it
in the
subset
if $x_3 = T$

use it
in the
subset
if $x_3 = F$
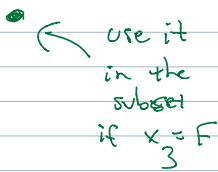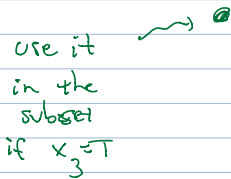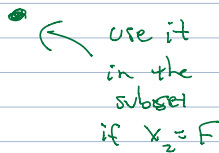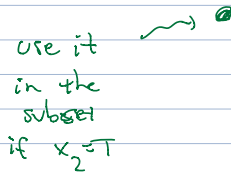
Size of subset, $A$, will be $a = 3$

Build graph:

use it
in the
subset
if $x_1 = T$

use it
in the
subset
if $x_1 = F$

enforce
each clause
is satisfied

0

0

0

8

etc.

$C_1$ vertex

use it
in the
subset
if $x_2 = T$

use it
in the
subset
if $x_2 = F$

similarly

use it
in the
subset
if $x_3 = T$

use it
in the
subset
if $x_3 = F$

similarly

Size of subset, A, will be $\boxed{a = 3}$

Enforce
$C_1 = (x_1 \text{ or } \neg x_2 \text{ or } \neg x_3)$

number of
vertices in
each piece
is $B$ (?)
(big but poly)

get $b$ neighbours for $b = B \cdot n + \# \text{clauses}$
if satisfiable