CPSC 421/501, Nov. 8

Now that any $L \in NP$ has $L \leq_p SAT$

and

$L \leq_p 3SAT$

Justify 3SAT

tricks!

show that also $L \leq_p SUBSET\text{-}SUM$

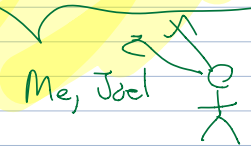
maybe also $L \leq_p PARTITION$

3CNF $\Leftrightarrow$ { u_1 OR u_2 OR u_3 OR u_4 OR u_5 } $= T$ i.e. it is satisfied

$\Leftrightarrow$ { $(u_1 \text{ OR } u_2 \text{ OR } d_1)$ and $(\neg d_1 \text{ OR } u_3 \text{ OR } d_2)$ and $(\neg d_2 \text{ OR } u_4 \text{ OR } u_5)$ is satisfiable }

Homework: $(x_{ijr} = true)$ and $(y_{ij} = true)$ and $(z_{iq} = true)$

$\Leftrightarrow$ $x_{ijr} \wedge y_{ij} \wedge z_{iq}$



Why 3SAT?

SUBSET-SUM:

$\{ \langle n_1, \ldots, n_m; t \rangle$ | Is there $I \subseteq \{1, \ldots, m\}$ s.t. $\sum_{i \in I} n_i = t \}$

a type of bin packing-like problem

e.g. given $\langle 3, 5, 7, 8, 12, 110, 56, 72; 171 \rangle$

if you could get ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ target ↑

which to take to sum to 171

① SUBSET-SUM $\in$ NP (easy)

② $L \in NP$, $L \leq_p$ SUBSET-SUM (tricky) } SUBSET-SUM is NP-complete

Proof: 3SAT $\leq_p$ SUBSET-SUM

Idea:

$f(x_1, \neg, x_n)$ in 3CNF

e.g. $(x_1 \text{ OR } x_2 \text{ OR } x_3)$ and $(\neg x_1 \text{ OR } x_2 \text{ OR } \neg x_3)$ and ...

create subset sum

n_1 pick if x_1 is T

n_2 " " x_1 is F

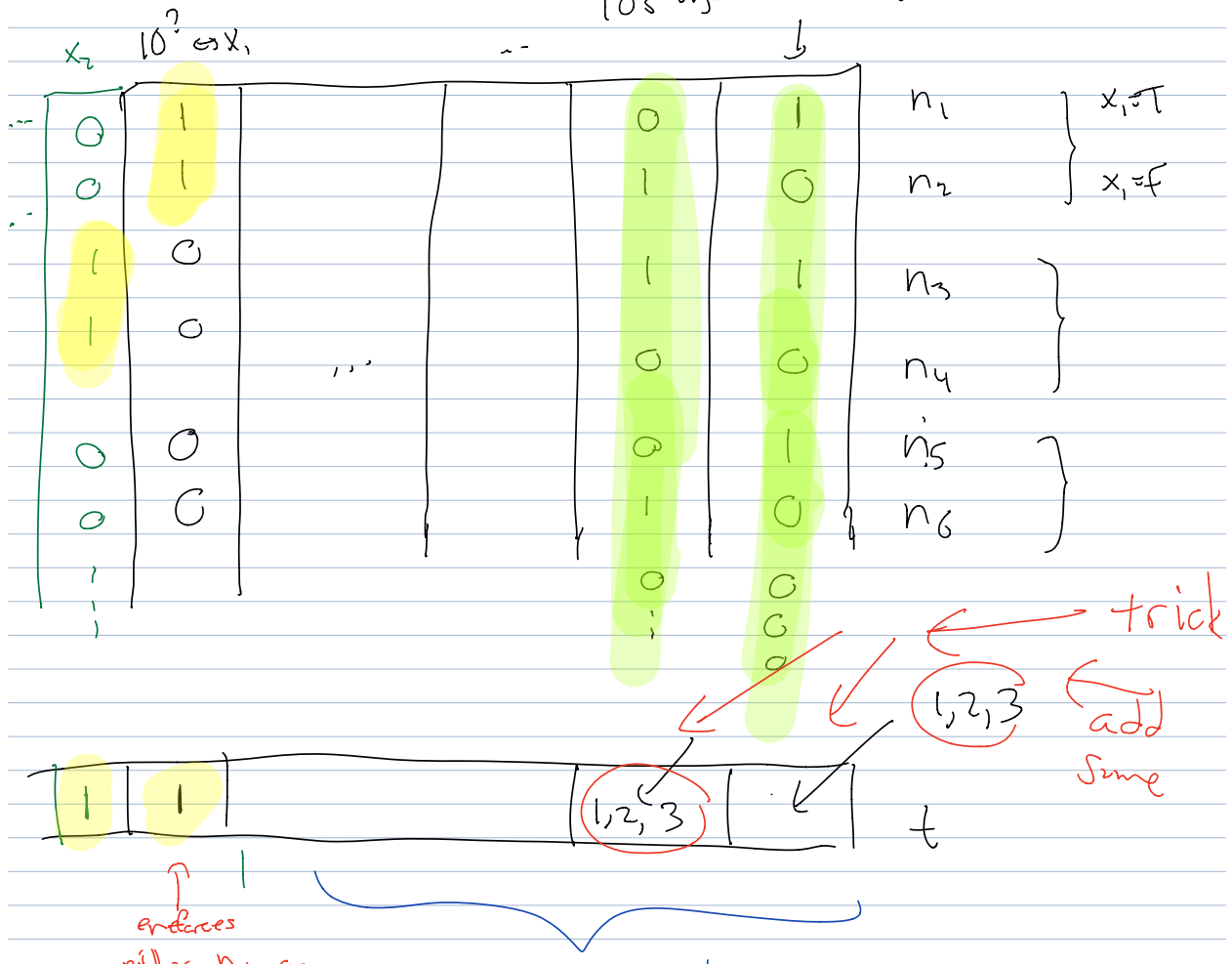
n_3 " " x_2 is T

n_4 " " x_2 is F

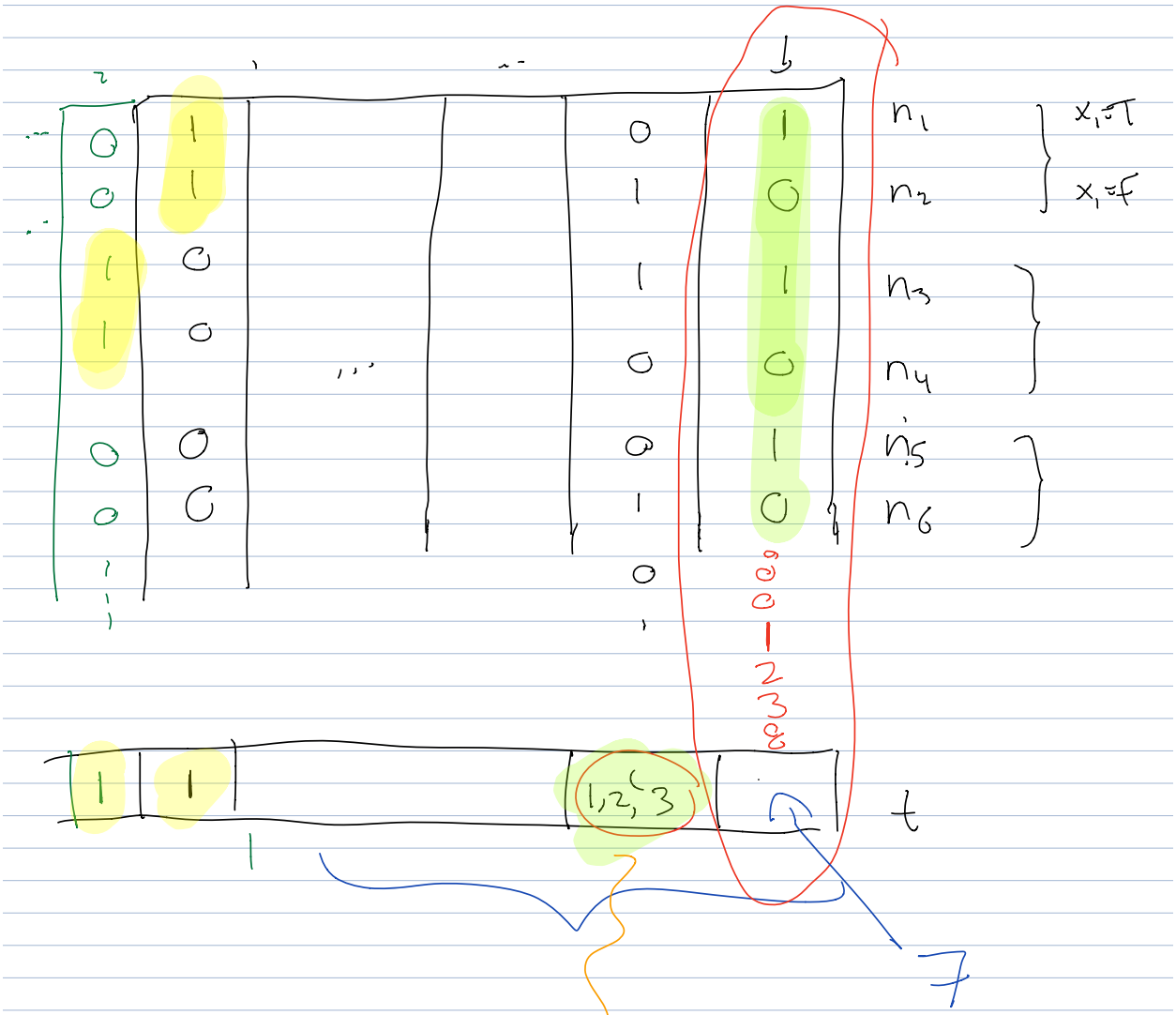
;

e.g. $(X_1 \text{ OR } X_2 \text{ or } X_3)$ and $(\neg X_1 \text{ or } X_2 \text{ or } \neg X_3)$

and — clause$_1$ ← c_1



Trick: turn 1,2,3 vs 0 into only one vs. 0 number



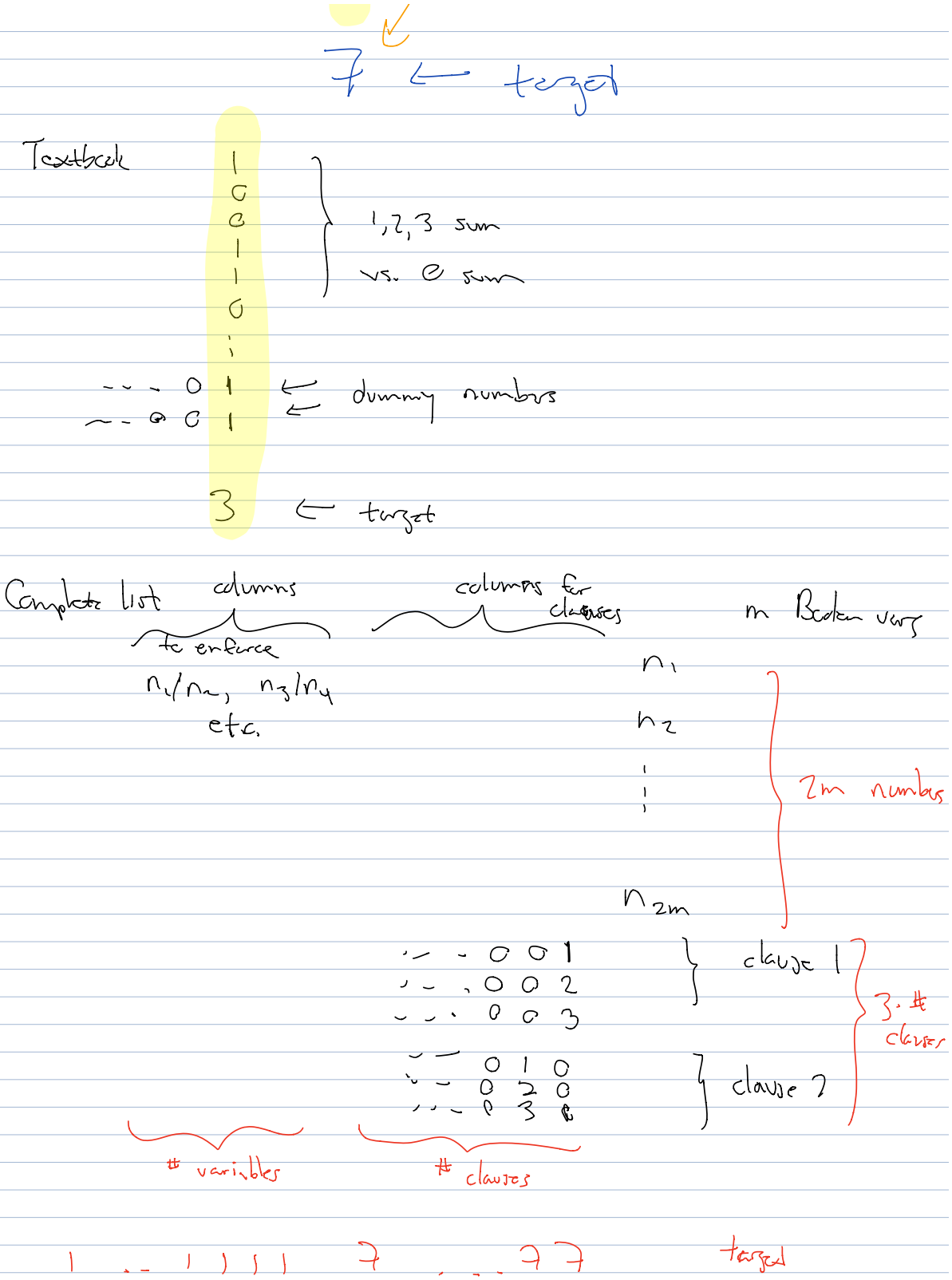
In detail

if have 1,2,3 (clause is true)

if have C, no way to get a 7,

"dummy numbers"

Important numbers to get the digit 7 to work out


7 ← target
Textbook
1,2,3 sum
vs. e sum
dummy numbers
3 ← target
Complete list
columns to enforce
n_1/n_2, n_3/n_4 etc.
columns for clauses
m Boolean vars
n_1
n_2
n_{2m}
2m numbers
clause 1
clause 2
3 · # clauses
variables
clauses
target