At last, a Turing machine that does something...

(computes a function)

Last time "$L_1 <_p L_2$"

"The language L_1 can be reduced in poly time to L_2"

Say have alphabets, Σ_1 and Σ_2, and languages L_1 over Σ_1 and L_2 over Σ_2.

We say that $f: \Sigma_1^* \to \Sigma_2^*$ can be computed in poly time if there is a Turing machine, M, that on input $w \in \Sigma_1^*$ runs in poly time, i.e. halts after at most $p(|w|)$ time for some poly p, and when M halts on input w, it writes on some designated tape $f(w)$.

input $\boxed{w_1 | w_2 | \dots | w_n | \sqcup | \sqcup | \sqcup} \dots \xrightarrow{\text{upon halting}} \underbrace{\boxed{w_1' | w_2' | \dots | w_n'}}_{f(w)} \sqcup \sqcup \dots$

We say L_1 can be reduced to L_2 in poly time, write $L_1 <_p L_2$ if $f: \Sigma_1^* \to \Sigma_2^*$ s.t.

1. f can be computed in poly time
2. for all $w_1 \in \Sigma_1^*$, $w_1 \in L_1 \iff f(w_1) \in L_2$

Today: ① Finish: If L_1 is any language in NP

(3COLOR, PARTITION, HAM-PATH, ...) then

$L_1 \leq_p$ SAT and $L_1 <_p$ 3SAT

② SAT $<_p$ SUBSET-SUM $<_p$ PARTITION

$<_p$ etc.

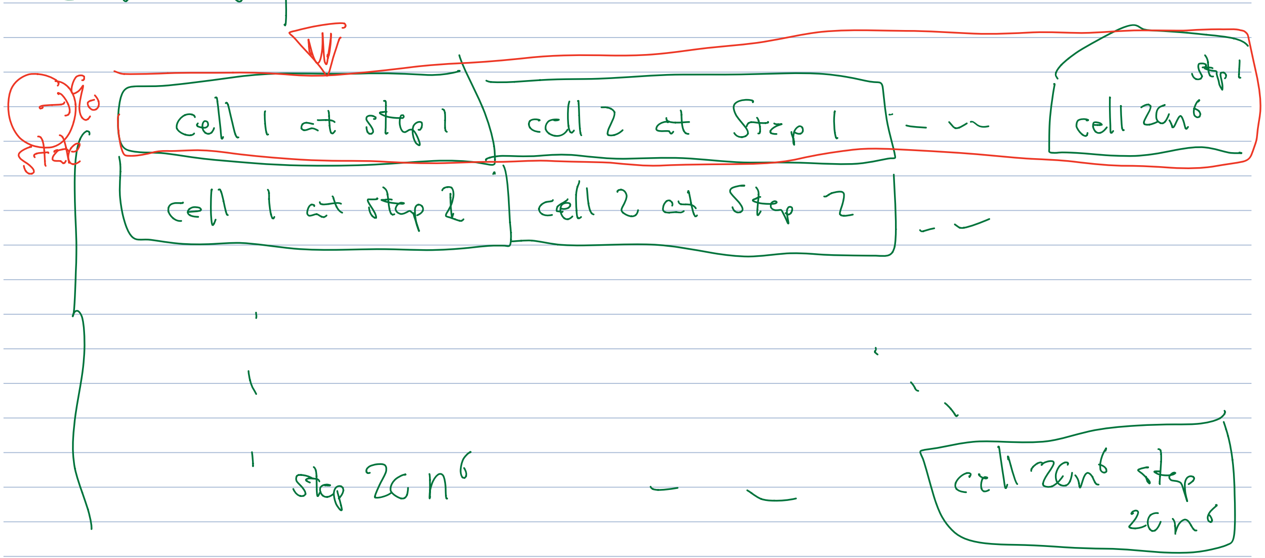
$<_p$ VERTEX-COVER

$<_p$ etc. $<_p$ MINE-SWEEPER

Fix a non-det Turing machine, M, say M runs in time $\leq 20n^6$.

Now given input to M $w = w_1 \dots w_n$, $n = |w|$

Intuitively



Step 1 : Need to be in state q_0 AND

Tape head is on cell 1

AND

On cell i, $i = 1, \dots, n$, symbol w_i is written

AND

At cell $i = n+1, \dots, 20n^6$

a $\sqcup$ has to be written

We introduced

$$X_{ij\gamma} = \begin{cases} T & \text{if at step } i \text{, cell } j \text{ symbol written is } \gamma \\ F & \text{otherwise} \end{cases}$$

$$Y_{ij} = \begin{cases} T & \text{if at step } i \text{, tape head is in cell } j \\ F & \text{---} \end{cases}, \qquad Z_{iq} = \begin{cases} T & \text{if at step } i \text{ we are in state } q \\ F & \text{---} \end{cases}$$

(1) Write out for each $i, j = 1, \dots, 20n^6$ that $X_{ij\gamma} = T$ for exactly one $\gamma \in \Gamma$ — poly time ☺

finite set

(1'), (1'') same for Y's, Z's

Step 1 : $Z_{1,q_0} = T$, $Y_{1,1} = T$, $X_{1\,1\,w_1} = T$,

$X_{1\,2\,w_2} = T, \dots, X_{1\,n\,w_n} = T$

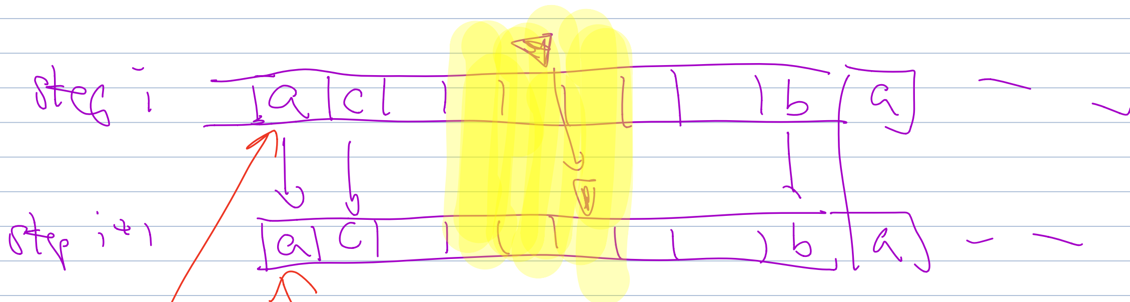
$X_{1\,n+1\,\sqcup} = T, \dots, X_{1\,20n^6\,\sqcup} = T$

$(z_{1,q_0}$ OR z_{1,q_0} or $z_{1,q_0})$ ← go roughly like

Step $20n^6$: $z_{20n^6, q_{acc}} = T$

For $i = 1, \dots, 20n^6 - 1$:

Step i → Step i+1 is a legal transition according to M



if is true, then

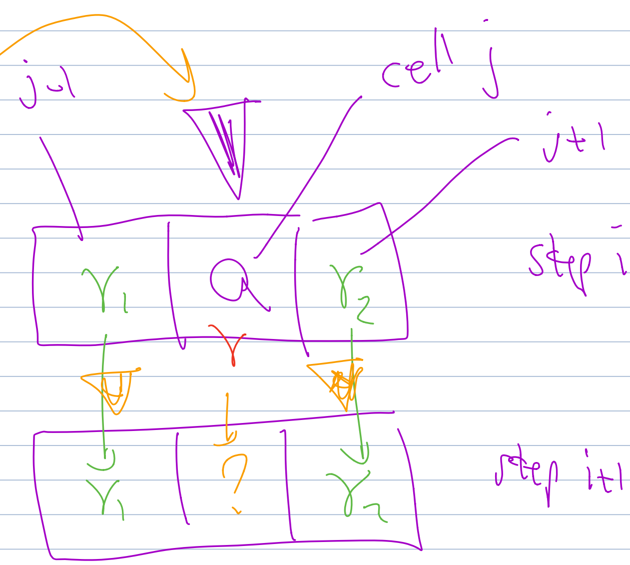
if ▽ is nowhere near you

state q ⟺

$z_{i,q} = T$

$x_{i,j,a} = T$

$y_{i,j} = T$



M tells you $\delta(q, \overset{\gamma_1}{a}) =$ etc.

Proves: $L_1 \in NP$, then $L_1 <_p 3SAT$

Next: $3SAT \underline{<_p}$ SUBSET-SUM

[Sip]

$<_p$ PARTITION

Given a 3SAT, i.e. a formula $f \in 3CNF$, want to know if f is satisfiable ...

SUBSET-SUM: $\{ \langle n_1, \ldots, n_s, t \rangle$ s.t. $\sum_{i \in I} n_i = t$ for some $I \}$

e.g.

$3, 4, 5, \underline{7}$: $3+4 = \underline{7}$ $\in$ SUBSET-SUM

$(X_1$ OR X_2 OR $X_3)$??

AND

⋮

$\longrightarrow$ SUBSET-SUM problem