- Midterm in $\boxed{\text{SWNG 121}}$
- My office hours tomorrow: 3-5pm, location TBA
- Midterms will be loaded into gradescope
    - Do not put any PI on the individual exam sheets
    - We will already label each sheet with you a1b2c-style old-style @ugrad.cs.... accounts
    - Please stay at ur room until we setup

Back to \$$10^6$ — solve P vs. NP ← { PARTITION, BINPACKING, SAT, 3COLOR }

Last time: this means: – Give polytime alg. for 3COLOR

OR – show no such alg exists...

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NP ⟷ Nondeterministic Polynomial Time

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Analogous to DFA vs. NFA

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Nondeterministic Turing Machine

$$\underbrace{\text{Turing Machine}}_{(Q, \Sigma, \Gamma, q_{init}, q_{acc}, q_{rej}, \sqcup, \delta)}$$

~~$\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$~~

$\delta : Q \times \Gamma \to \text{Power}(Q \times \Gamma \times \{L, R\})$

you can have more than
computation path / branch

We say a Non-det TM, M, runs in time $f(n)$, for $f : \mathbb{N} \to \mathbb{N}$ ($\mathbb{Z}_{\geq 0} \to \mathbb{Z}_{\geq 0}$)

if on input $w \in \Sigma^n$, all possible computations paths halt within $f(n)$ steps. If so M decides the language

$$\left\{ w \in \Sigma^* \;\middle|\; \begin{array}{l} \text{there is at least one} \\ \text{accepting path for } M \\ \text{on input } w \end{array} \right\}$$

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Claim: 3COLOR can be decided in Non-det poly time, i.e. there is a Non-det T.m., M, runs in time $O(n^k)$ for some k that decides 3COLOR:

why: input: $\langle G \rangle$ → 20 # 1 # 2 # 1 # 17 # 2 # 4 .... etc.

tape 1

On tape 2 write a string of 20 characters R (red), G (green), B (blue)

Start with $\langle G \rangle$

cell 1 write tape 2: R G B

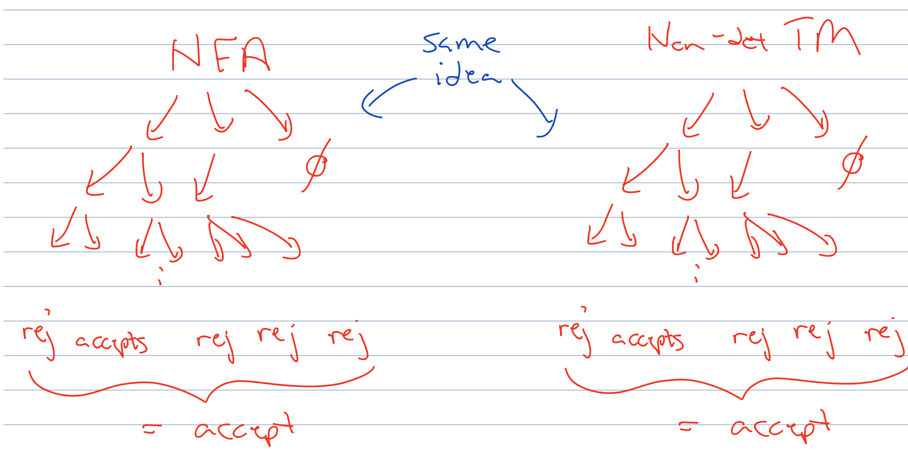
cell 2 tape 2: R G B

tape 2: RGRBGRRR...R ␣␣␣

Phase 1

Phase 2: Check if this colouring is a proper 3-colouring of G on input tape.

⇒ Then, by definition of non-det T.M., we are done ☺



time = max # of steps in any computation path

⇒

SAT = { $\langle f \rangle$ | Boolean formula that has a satisfying assignment } e.g. $(x_1 \wedge \neg x_2) \vee (x_1 \wedge x_2)$

Midterm practice...

Say $f(n) = n+3$, $f: \mathbb{Z} \to \mathbb{Z}$

Prove f is surjective: given an $m \in \mathbb{Z}$, there is an $n \in \mathbb{Z}$ s.t. $f(n) = m$: if $n \leftarrow m-3$

$f(n) = n+3 = m-3+3 = m$.

Representing T.m:

- Once you set $Q = \{1, \dots, q\}$, $\Gamma = \{1, \dots, r\}$ $\Longrightarrow$ countably many TMs

- # T.m. is vastly uncountable, but $\Longrightarrow$ countably many algorithms

T.m. $\overset{\text{rep. as}}{\Longleftrightarrow}$ finite strings

Graph $\underset{\text{rep. as}}{\Longleftrightarrow}$ " "

Proofs by diagonalization:

$f: S \to \text{Power}(S)$ then

$T = \{ s \in S \mid s \notin f(s) \}$ is not in the image of f.

$S = \{1,2,3\}$, f: $1 \mapsto \emptyset$, $2 \mapsto S$, $3 \mapsto \{1,3\}$

$$T = \begin{bmatrix} 1 \\ \cancel{2} \\ \cancel{3} \end{bmatrix}$$

$1 \notin \emptyset = f(1)$

$2 \in S = f(2)$

$3 \in \{1,3\} = f(3)$

Remark: In Ch. 3, Midterm, only going to use

T.m. that $\boxed{\text{decide}}$ a language:

M decides a language, L, if M halts on all inputs

then $w \in L \Rightarrow$ M reaches q_{acc}

$w \notin L \Rightarrow$ " " q_{rej}