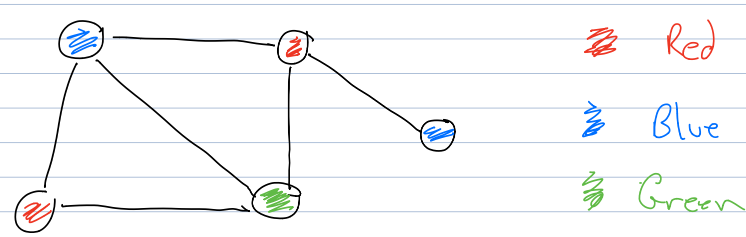
How to get a quick $1,000,000 ...

possible

Ch 7 [Sip]: Solve:

$$3COLOR = \{ \langle G \rangle \mid \text{G is graph that can be } \underbrace{\text{3-coloured}} \}$$

i.e.



Formally: $G = (V, E)$, we say G is 3-colourable if $V \to \{\ ,\ ,\ \}$ s.t. each edge has its endpoints coloured differently.

For $\$10^6$: Either (1) Show that 3COLOR can be decided in poly time.
(2) Show that it can't be.

§7.1 & 7.2 Formalize this ...

Really: 3COLOR, SAT $= \{ \langle f \rangle \mid$ f is a Boolean formula that can be satisfied $\}$, many more & if you can solve of these in poly time, you can solve them all. And ... NP to conceptualize these problems.

Formalize: The *time* that a Turing machine, M, takes on input w is the number of steps to reach q_{acc} or q_{rej}.

We say that M runs in time $f(n)$, where $f: \mathbb{N} \to \mathbb{N}$ or $\mathbb{Z}_{\geq 0} \to \mathbb{Z}_{\geq 0}$

if on any input w of length $n = |w|$, the Turing machine takes time at most $f(n)$.

"linear time algorithm" time $O(n)$

"quadratic time algorithms" time $O(n^2)$

etc.

← RAM

Random access machine

Avoid

We say $f: \mathbb{N} \to \mathbb{N}$ is $O(g(n))$ for another $g(n)$ if for some C and n_0

$$f(n) \leq C\, g(n) \qquad \text{if } n \geq n_0$$

E.g. $3n^2 - 2n + \log n = O(n^2)$

$3n^2 - 2n + \log n = O(n^{1000})$

"linear time" ↔ $O(n)$ time

"quadratic time" ↔ $O(n^2)$ time

polynomial time ⟷ for some $k \in \mathbb{N}$, $O(n^k)$

P = languages decidable in polynomial time = languages decidable in time $O(n^k)$ for some $k \in \mathbb{N}$

Should be convinced that a lot of practical algorithms run in poly time ... But poly time can mean time $10^{10^{10^{10}}} \cdot n^2 = C n^2$, $C = 10^{10^{10^{10}}}$

$=$

$OO(n^2) =$ Udi Manber Uh-oh of n^2

$=$

But poly time is a reasonable class to consider

$O(n^{1000000})$ maybe $\leq 3 \cdot n^{1000000}$

$=$

NP = Non-deterministic poly time ...

Why
3COLOR
SAT
⋮
are interesting...

$=$

Midterm practice:

δ for a Turing Machine, or DFA:

Algorithm: Phase 1 / Phase 2 } Give δ values OR state diagram

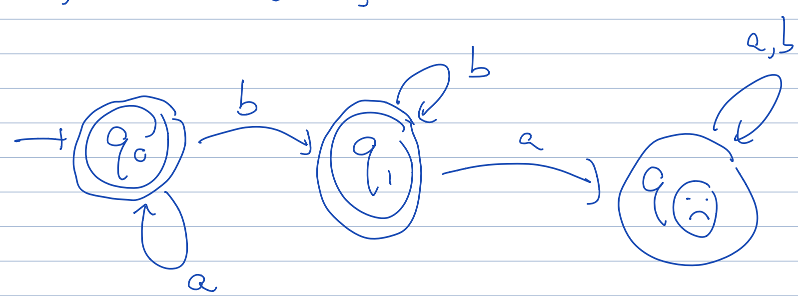
$$\text{T.M.} = (\underbrace{Q}_{}, \underbrace{\Sigma}_{\text{given}}, \underbrace{\Gamma}_{\text{at least } \Sigma \cup \{\sqcup\}}, \underbrace{q_{acc}, q_{rej}, q_{init}}_{\text{usually clear}}, \sqcup, \delta)$$

q_{init}: q_0

=

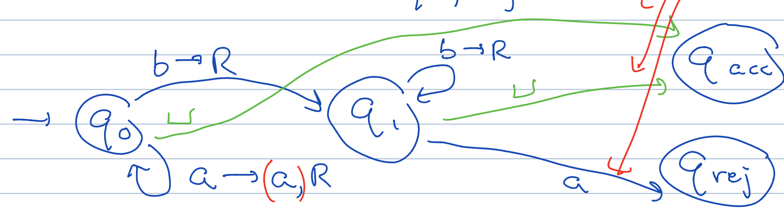
$a^* b^*$, $\Sigma = \{a, b\}$

DFA



TM: aaabbb⊔⊔⊔⊔

$\delta_{TM}: Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$



when transitioning to q_{acc}, q_{rej} don't need to specify ~~what~~ what you write, where you move.

$a \to a, R$ can abbreviate $a \to R$

Also $\begin{array}{l} a \to a, L \\ c \to c, L \end{array}$ abbreviate $a, c \to L$

$\underbrace{1312111}_{x}, 8 \longrightarrow 10x + 8$

$(10x + 8) \bmod m$

$= (x \bmod m) \cdot 10 + (8 \bmod m)$

Mostly 3.3 for us

$\langle G \rangle$ G = graph

$\langle M, w \rangle$ M = T.m., w = input