Recognizable (typically by "simulating" or with a Universal TM.)

but undecidable:  

- $\overline{A_{TM}}$, $\overline{Halt_{TM}}$, $\overline{DoWeEnterACertainState_{TM}}$
- Does TM Accept Any Strings At All, Do Two TMs Recognize Different Languages, ...
- Does A TM Halt On Any Input At All

Start Ch 7: Poly Time

Do Two TMs Recognize Differently?

$L_2 = \{ \langle M_1, M_2 \rangle \mid \text{for some input there is a string accepted by both} \}$

Claim: $L_2$ is undecidable.

$L_2 = \{ s \in \Sigma^* \mid \text{for some input } s = \langle M_1, M_2 \rangle \text{ and } \}$

Imagine (for the sake of contradiction) that $L_2$ is decidable.

Then I build an algorithm to solve $A_{TM}$ using the algorithm for $L_2$:  


Say given \( \langle M, w \rangle \) and we want to know if \( \langle M, w \rangle \in A_{TM} \), i.e. if \( w \) is accepted by \( M \).

**Method 1:** Build a machine \( M_2 \) that accepts \( w \) and only \( w \)

Then feed \( \langle M, M_2 \rangle \) into \( L_2 \) algorithm:

\[ L_2 \text{ algorithm (simplifies)} \]

says "yes" iff \( M \) accepts \( w \).

**Method 2:** From \( M, w \) let's build a \( TM \) that erases input.

1. Write \( w \) on tape
2. Run \( M \) on it

So \( \hat{M} \) accepts \( w \) if \( \langle M, w \rangle \in A_{TM} \), otherwise

\[ \hat{M} \text{ rejects } w \]

Feed \( \langle \hat{M}, \hat{M} \rangle \) to \( L_2 \), \( L_2 \) "yes" if \( \langle M, w \rangle \in A_{TM} \)

\[ \setminus \]

Accept Empty String \( \langle M \rangle \)

\[ \text{ACCEPT - EMPTY STRING } \text{ } TM = \{ \langle M \rangle \mid \text{ } M \text{ is a Turing machine and } \text{ } M \text{ accepts } \epsilon \} \]

is undecidable:

\[ \not\exists \]

If \( L_3 \) is decidable, given

\[ \langle M, w \rangle \quad \longrightarrow \quad \text{build } \hat{M} \quad \text{feed } \hat{M} \quad \text{into } L_3 \]

in \( L_1 : A_{TM} \)

Start Poly Time (Ch 7) on Friday.