CPSC 421/501

# MIDTERM IN SWNG 121

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Recognizable (typically by "simulating" or with a Universal T.m.)

but undecidable: $A_{TM}$, $Halt_{TM}$, DoWeEnterAGivenState$_{TM}$,

New DoesTMAcceptAnyStringsAtAll, DoTwoTMsRecognizeDiffLangs, ...

DoesATMHaltOnAnyInputAtAll, ...

New

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StartCh7: PolyTime

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~~DoTwoTMsRecognizeDiffLangs~~?

DO-TWO-TMs-HAVE-A-STRING-THAT-THEY-BOTH-ACCEPT ? $= \{ \langle M_1, M_2 \rangle \mid M_1, M_2$ are Turing machines s.t. there is a string accepted by both $\}$

$L_2$

$L_1 \rightarrow A_{TM}$ ← We know $A_{TM}$ is undecidable

Claim: $L_2$ is undecidable.

$L_2 = \{ s \in \Sigma^* \mid$ s.t. $s = \langle M_1, M_2 \rangle$ and $\}$

Imagine (for the sake of contraction) that $L_2$ is decidable.

Then I build an algorithm to solve $A_{TM}$ using the algorithm for $L_2$:

Say given $\langle M, w\rangle$ and we want to know if $\langle M,w\rangle \in A_{TM}$ i.e. if $w$ is accepted by $M$.

Method 1: Build a machine $M_2$ that accepts $w$ and only $w$

builds a mess 102#37#38#...

Then feed $\langle M, M_2\rangle$ into $L_2$ algorithm: $L_2$ T.M. algorithm (subroutine) says "yes" iff $M$ accepts $w$.

Method 2: From $M, w$ let's build a TM $\widetilde{M}$ ← built on $M$ and $w$ ①: erases input

② write $w$ on tape 1 ③ run $M$ on it.

So $\widetilde{M}$ accepts $\Sigma^*$ if $\langle M,w\rangle \in A_{TM}$, otherwise

$\widetilde{M}$ { rejects $\Sigma^*$ / loops $\Sigma^*$

Feed $\langle \widetilde{M}, \widetilde{M}\rangle$ to $L_2$, $L_2$ "yes" iff $\langle M,w\rangle \in A_{TM}$

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$L_3$

$AcceptEmptyString_{TM}$

$ACCEPT\text{-}EMPTY\text{-}STRING_{TM} = \{ s \mid s = \langle M\rangle, M \text{ is a Turing machine, accepting } \varepsilon, \text{ and possibly other stuff}\}$

is undecidable:

Pf: If $L_3$ is decidable: given

$\langle M, w\rangle$ in $L_1 = A_{TM}$ → build $\widetilde{M}$ as above → feed $\widetilde{M}$ into $L_3$

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Start Poly Time (Ch 7) on Friday