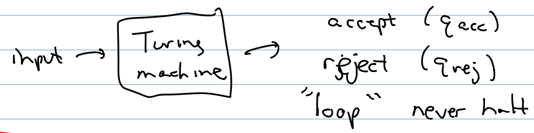
Today: § 4.2 + § 5.1 (in part)

Exam: One 2-sided $8\frac{1}{2} \times 11$ sheet of notes,

Textbook: Formal description: you specify δ

Implementation level: you gives phases, how many tapes, how they move, etc.

High level: algorithm without any of discussion of tapes



Only Countably many "standard T.M.'s" $Q = \{1, \ldots, q\}$ $\Gamma = \{1, \ldots, r\}$

$A_{TM} = \{ \langle M, w \rangle \mid M \text{ is a Turing machine that accepts } w \}$ is undecidable.

Proof that A_{TM} is undecidable: If H decides A_{TM} (assume to get a contradiction), then build a Turing machine, using H as a subroutine, s.t. this Turing machine can't exist.

↕ Oracle Turing machines

Build a machine D s.t.

(1) given input w, D figures out if $w = \langle M \rangle$ for some Turing machine.

Recall: Turing machine description was based on $\Sigma = \{0, \ldots, 9, L, R, \#\}$

But Σ really should be $\{0, \ldots, 9, 10, 11, 12\}$

D { ② If $w = \langle M \rangle$, feed $\langle M, \langle M \rangle \rangle$ to H (i.e. run H as a subroutine.

③ If H accepts $\langle M, \langle M \rangle \rangle$, then D rejects ← Univ T.M

" " rejects $\langle M, \langle M \rangle \rangle$, then D accepts

[" " doesn't halt ← can't happen ←]

What happens to D on input $\langle D \rangle$?

If D accepts $\langle D \rangle$, then H rejects $\langle D, \langle D \rangle \rangle \Rightarrow$ D reject $\langle D \rangle$

Remark: A_{TM} is recognized by a Univ Turing machine

But $\overline{A_{TM}} = \Sigma^* \setminus A_{TM}$ is not even recognized by any Turing machine

Why: Say you can recognize L by a Turing machine M_1

and " " " $\overline{L}$ " " " " M_2

Given w, can run in parallel M_1 and M_2

A lot of this course: the following are undecidable:

A_{TM}, $Halt_{TM}$, DoYouReachState, ...

which problems should we not work too long to completely solve

the following are NP-complete

SAT, 3COLOR, ...

the following are PSPACE-complete

...

Say $L = \{ \langle M, w, q \rangle \mid$ M is Turing machine, w input to M, we do encounter state q along the computation of M given q $\}$

= DoWeReachSomeState

= DO-WE-REACH-SOME-STATE is recognizable, but undecidable

If we simulate M on input w, then if we ever land in q, we do so in finite number of steps, so we can stop and say "yes."

But if L were decidable, to solve A_{TM} we just ask given $\langle M, w \rangle$ does $\langle M, w, q_{acc} \rangle \in L$

Given $L_1, L_2, \ldots$ are recognizable, then $L_1 \cap L_2$, $L_1 \cup L_2, \ldots$ are also recognizable