CPSC 421/501

- Midterm on Wednesday, Oct. 30
- Some study materials are available online
- HW 6: Not collected, solutions will be up on Oct 23.

M	W	F
Most of the class we will take question → 28	30	25 ← latter ½ class take question

- Exam coverage is on website
  - Handout on countability etc
  - Ch 1 [Sip]: Added Myhill-Nerode
    Omitted §1.4 Pumping Lemma
  - Ch 3 [Sip]: 3.1 Turing machines
    ~~3.2 Variants~~
    Only ask for Turing construction for regular language
    3.3 = ~~Hilbert's Problems~~ Descriptions of graphs, T.m's, DFA

$\langle G \rangle$
$\langle M \rangle$
$\langle D \rangle$

$\langle M, w \rangle$ = description of Turing machine + input

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DFA, Turing machine
NFA

Phase 1: blah
Phase 2: -
Phase 3: -

then implement

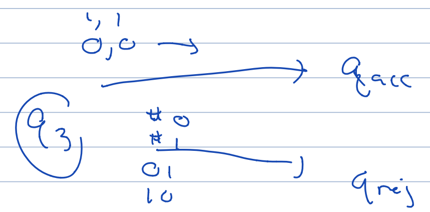
q_0, q_E, $q_{\vdots}$

Sometimes

$u = 0,1,\#,\ v = 0,1$

$$\delta(q_3, u, v) = \begin{cases} q_{rej} & u \neq v \\ q_{acc} & u = v \end{cases}$$

might be easier to write than



Universal Turing machines:

M Turing machine, w input to M

(insist state = {1, ..., q}
$\Gamma = \{1, \dots, \gamma\}$

then $\langle M, w \rangle$ = 23# 3 #400 # $\delta(1,1)$ # $\delta(1,2)$ ⋮ # $\delta(q, \gamma)$ ☺ 1#2#2#2#2#2#2#2 #3

(annotations: q over 23; size of $|\Sigma|$ over 3; r over 400; "gives state, tape symbol, L/R" over $\delta(1,1)$ → 5#397#R; $Q = \{1, 2, \dots, 23\}$, convention: $q_{init} = 1$; input → ☺ 1#2#2#2#2#2#2#2 #3)

long word over $\{0, \dots, 9, \#, ☺, L, R\}^*$

Claim: There is a universal Turing machine, i.e.

on input $\langle M, w \rangle$ we end on q_{acc} if M accepts w

q_{rej} " " rejects w

(loop) never halt " " never halts on w

Rem: There isn't a universal DFA ... (Comparison)

" " " " NFA

Claim: $Accept_{TM} = A_{TM} = \{ \langle M, w \rangle \mid M \text{ accepts } w \}$

is recognizable but undecidable...

Same $Halt_{TM} = \{ \langle M, w \rangle \mid M \text{ accepts } w \text{ or } M \text{ rejects } w \}$

Decidable: M decides a language L over Σ,

for all $w \in \Sigma^*$, $w \in L$ M accepts w

$w \notin L$ M rejects w

Recognizable Any Turing machine M with input alphabet Σ

recognizes $L = \{ w \in \Sigma^* \mid M \text{ accepts } w \}$.

(It doesn't include $w \in \Sigma^*$ s.t. M rejects or loops on w.

Example: A universal Turing machine recognizes A_{TM}.

So A_{TM} is recognizable by a Turing machine (a universal TM)

Claim: A_{TM} is not decidable.

Proof: Say that H decides A_{TM}. Then build another

T.M: ① on input S, see if S is the description $\langle D \rangle$

of a valid Turing machine.

M { ② if so, "feed" $\langle D, \langle D \rangle \rangle$ to H
③ if H accepts $\langle D, \langle D \rangle \rangle$ we reject D
if H rejects $\langle D, \langle D \rangle \rangle$ we accept D

What happens to M on input $\langle M \rangle$???

Last: Complement of A_{TM}, $\overline{A_{TM}}$ is not recognizable