CPSC 421/501 Oct 16, 2019

Thm 4.11 (Section 4.2): $A_{TM} = \left\{ \langle M, w \rangle \;\middle|\; \begin{array}{l} M \text{ is a Turing machine} \\ w \text{ is accepted by } M \end{array} \right\}$

$\underbrace{\text{(is recognizable)}}_{\text{universal Turing machine ☺ !!!}}$ but not decidable.

(1) Started: $L = \{0^n 1^n \mid n \in \mathbb{N}\}$ is Turing decidable

Decidable: A language is Turing-decidable if there is a T.M., M, that on input $w \in \Sigma^*$, M accepts w if $w \in L$

M rejects w if $w \notin L$

Remark: A Turing machine on a given input can

(1) accept the input — it reaches q_{accept} (finite number of steps)

(2) reject " " — " " q_{reject} (" " " ")

(3) "loops" or "does not halt", i.e. runs forever, never reaching q_{acc} or q_{rej}

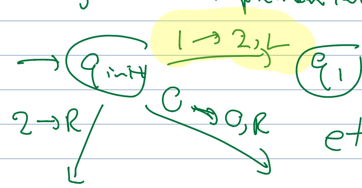
(2) PALINDROME is Turing decidable

To describe a T.m., you usually

- give a high level description
- " a medium level " (describes how many tapes, various phases, etc.)
- give an "implementation"

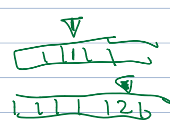
Specify δ:

$\delta(q_{init}, 1) = (q_1, 2, L)$



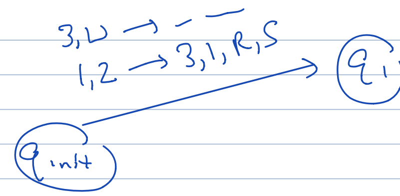
etc.

Say 2-tape



$\delta(q_{init}, 1, 2) = (q_1, 3, 1, R, S)$

tape 1 tape 2



$\delta: Q \times \Gamma^2 \to Q \times \Gamma^2 \times \{L, R, S\}^2$

$\delta(q_{init}, x, y) = (q_{init}, x, y, R, S)$ if $x \neq \sqcup$ and y any

med-level
low-level implementation : $\{0^n 1^n\}$, PALINDROME, Homework, $\{0^{2^n} \mid n \in \mathbb{N}\}$

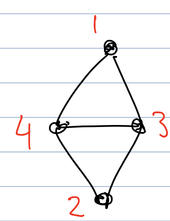
Next 2 examples in Ch 3 — textbook skips the implementation

Examples in 4.1 & 4.2

In 4.2, the textbook describes a universal Turing machine in one sentence

Warm up! Graph (V, E), ⟨graph⟩ — give # vertices, a list of edges

vertly infinitely uncountably many



4 # 1 # 3
3 # 4
4 # 2
3 # 2
1 # 4

maybe

4 # 1 # 3 # 1 # 4 # 2 # 3 # 2 # 4 # 3 # 4

(lexicographical order to get a unique representation)

① Let $L = \{\langle G \rangle \mid G$ is a graph that can be 3-coloured, i.e. there exists : $V \to \{1,2,3\}$ s.t. each edge has different colours $\}$

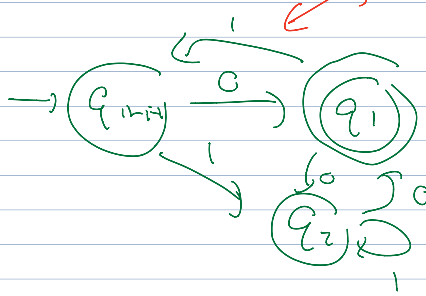
started to give an algorithm

tape 2 for bleh
type 3 for bleh bleh
⋮

② Let $L = \{\langle D, w \rangle \mid D$ is a DFA, w is an input to the DFA that is accepted by the DFA $\}$

" A_{DFA}

describe



q_{init}	1
q_1	2
q_2	3

, describe δ, F, q_{init},

describe input,

$Q \longleftrightarrow \{1, \ldots, q\}$

$q_{init} = 1$, $\Sigma = \{1, \ldots, \sigma\}$, ...

Section 4.1 of (Sipser)

A_{DFA} = inputs accepted by DFA

③ Last Friday: $\langle M, w \rangle$ description of a T.m. and w input

$(Q, \Sigma, \Gamma, \delta, q_{acc}, q_{rej}, \sqcup, q_{init})$

27 # 3 # 5 # long description of δ $\delta(1,1)\#$ $\delta(1,2)\#$ ⋮   # 2 # 1 # 3

Claim: There is a "universal Turing machine" U, s.t.

on input $\langle M, w \rangle$, U stops in q_{acc} if M accepts w

U stops in q_{rej} " " rejects w

time ??? → ... U doesn't halt if M doesn't halt on input w

(Ch 7: poly time)

"debugger"

Describe: input describe M describe w

tape 1 → $|Q| \# |\Sigma| \# |\Gamma| \#$

$\# \delta(1,1) \# \delta(1,2)$

$\# \ldots$

$\# \delta(q,r)$

☺ $\sigma_1 \# \sigma_2 \# \ldots \# \sigma_n$

$(\sigma_i \in \{1, \ldots, |\Sigma|\})$

to say when input starts

tape 2 → copy input

(simulation of the T.m. tape) $\textcircled{T} \sigma_1 \# \sigma_2 \# \ldots \# \sigma_n \sqcup \sqcup \sqcup$

$\textcircled{T} \in \Gamma$

where the tape head is

tape 2 future $\gamma_1 \# \gamma_2 \# \underbrace{\textcircled{T} \gamma_3}_{\text{look here}} \# \gamma_4 \# \ldots$ etc.

look