Homework : make sure (1) you explain your solution
(2) all handwriting is legible
(3) your solution is concise

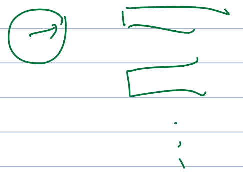
Today: $\langle \ \rangle$, deciding, halting, recognizing.

Cheating with $\langle \ \rangle$

What is an algorithm ? Answer : DFA/NFA limit algoriths
Turing Machines good general algorithm

$L = \{0^n 1^n\}$ can be recognized by a Turing Machine

What if Turing Machine with countably infinite number of tapes



(1) If you promise me that your algorithm will only use finitely many tapes for "the algorithm" the rest ~~for~~ for storage, then OK.

(2) The literal: $\delta : Q \times \Gamma^{\mathbb{N}} \to Q \times \Gamma^{\mathbb{N}} \times \{L,R,S\}^{\mathbb{N}}$ is too powerful

§3.3 What is an algorithm?

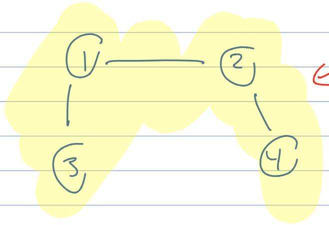
Graph --- { is it connected ?
is it 3-colourable ?
⋮

So we need to describe each graph (inputs) as string over some alphabet Σ.

Want to describe Turing machines, list to sort, etc.

$\langle \ \rangle$ a fixed description:

Graph : (V,E)



number of vertices

4 #1#2
#2#4
#1#3

4#1#2
#1#3
#2#4

$\langle$Graph$\rangle$ = Assume vertices numbered 1,..., n

Describe edges as pairs

$\Sigma = \{0, \ldots, 9, \#\}$

$\langle$Graph$\rangle$ the unique description

sometimes uniqueness is not an issue

sometimes it is ...

Can we check, given $\langle G \rangle$, if G is 3-colourable, with a Turing machine?

Input |4|#|1|#|2|#|2#4|#|1|#|3|⊔|⊔|⊔|⊔ ...

High level description: Read # vertices. Have a list of colours 1,2,3

4 vertices

|1|1|1|1|⊔|⊔|⊔|⊔|

tape 2

cycle through

1	1	1	2
1	1	1	3
1	1	2	1

any value

|3|3|3|3|⊔|⊔|- -

for each value of 1,2,3 on each vertex

check if it is a valid 3 colouring

|1|2|1|3|⊔ ⊔

Can you take $\left\{ \begin{array}{l} \text{Javascript} \\ \text{Python} \\ \text{C} \\ \text{Fortran} \\ \text{Algol} \\ \text{APL} \end{array} \right\}$ and convert to a Turing machine algorithm?

Ans: Yes, yes with a Turing machine.

=

Given a Turing machine, and input, can you see what happens as you run the T.m. on the input?

$$\langle \underset{\text{Turing machine}}{M}, \underset{\text{input to the Turing machine}}{i} \rangle$$

$$\text{T.m.} = M = (\underset{\text{some set}}{Q}, \Sigma, \Gamma, q_0, q_{acc}, q_{rej}, \sqcup, \delta)$$

Assume: $Q = \{1, 2, 3, \dots, q\}$

“standardized Turing machine”

$\Sigma = \{1, \dots, \sigma\}$

$\Gamma = \{1, \dots, \sigma, \underset{\sqcup}{\sigma+1}, \dots, \gamma\}$

$$\delta : \underset{\{1,\dots,q\}}{Q} \times \underset{\{1,\dots,\gamma\}}{\Gamma} \to Q \times \Gamma \times \{L, R\}$$

δ consists of $\underbrace{\delta(1,1)}_{\substack{\text{some state} \\ \text{some elt } \Gamma \\ L, R}}, \delta(1,2), \dots, \delta(1,\gamma), \delta(2,1), \dots$

$-- \ \delta(q,\gamma)$

Let $A = \{0, \dots, 9, \#, L, R\}$: write down each T.m.

$q \# \sigma \# \gamma \# \delta(1,1) \longleftarrow$ some state number # some tape symbol # ↖ L or R

$\# \delta(1,2)$

$\vdots$

Example $12 \# 3 \# 17 \# \delta(1,1) \leftarrow 9 \# 16 \# L \#$

$\delta(1,2) \leftarrow 11 \# 7 \# R \#$

$\vdots \quad \vdots$

Input $\Sigma = \{1,2,3\}$

$1 \# 2 \# 1 \# 3 \# 2 \# 2 \# 3$

Describe $\langle M, i \rangle$ string in $\{0, \ldots, 9, \#, L, R\}^*$