Section 1.3 Regular Expressions

- Regular Expressions (Limited Set)
- Equivalent to Regular Languages

Regular Expression: over alphabet Σ:

(1) $\{\sigma\}$ $\quad \sigma \in \Sigma$

(2) (ε) $\quad \varepsilon$ = empty string

(3) $\emptyset$ $\quad$ empty set

If R_1, R_2 regular expressions, then

(4) $(R_1 \cup R_2)$ is a regular expression

(5) $(R_1 \circ R_2)$, or simply $(R_1 R_2)$ is a regular expression

(6) (R_1^*) is a regular expression

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e.g. $\Sigma = \{a, b\}$

- Then $(a \cup b)^* b$ is a regular expression

- Express two-letter words: $\underbrace{(a \cup b)}_{\Sigma} \circ \underbrace{(a \cup b)}_{\Sigma}$ ← shorthand

$\Sigma = \{a, b\}$

$\Sigma \circ \Sigma$

$\Sigma^2 = \Sigma \circ \Sigma$ ← shorthand

- Express single word a^5

$((a \circ a) \circ (a)) \circ (a \circ a)$ write as a^5

$a \circ a \circ a \circ a \circ a$ write as a^5

$\Sigma = \{0, 1, \ldots, 9\}$

Refer to $\{1,3,5,7,9\} \leftrightarrow (((1 \cup 3) \cup 5) \cup 7) \cup 9)$ 

$1 \cup 3 \cup 5 \cup 7 \cup 9$

$1, 3, 5, 7, 9 \qquad \cup \leftrightarrow ,$

Given regular expression, R, there is an associated language $L(R)$.

$(a^5 \cup a^7)^*$, $L((a^5 \cup a^7)^*) = \{\varepsilon, a^5, a^7, a^{10}, a^{12}, a^{14}, \ldots\}$

Sometimes

$L^+ = L^1 \cup L^2 \cup L^3 \cup \ldots = L L^*$ = { one or more elements of L concatenated together }

So we allow $L((abb)^+) = \{abb, abbabb, \ldots\}$

Thm: L is regular $\iff$ there is a regular expression describing L

Textbook: $\emptyset^* \stackrel{?}{=} \{\varepsilon\} = \emptyset^0$

$= \emptyset^0 \cup \emptyset^1 \cup \ldots$

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Easy! Given a regular expression, R, we can draw an NFA that recognizes the language R describes.

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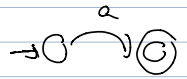
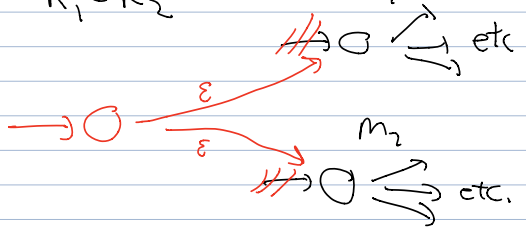
Why: $\Sigma = \{a, b\}$ regular expression a 

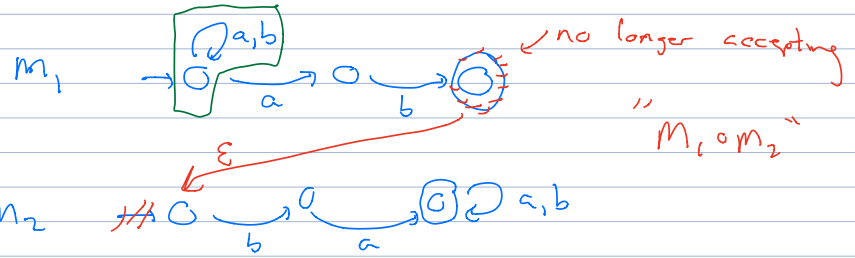
Given R_1, R_2 to each NFA, M_1, M_2 | ε $\emptyset$

$R_1 \cup R_2$ $R_1 \circ R_2$ R_1^*

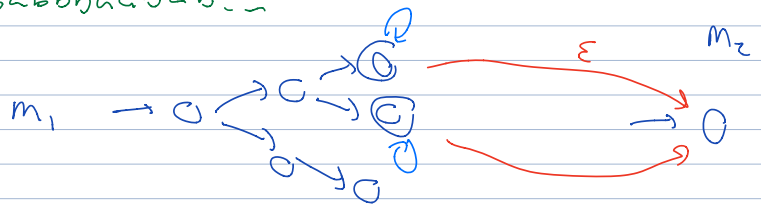


$R_1 \circ R_2$: $R_1 = \Sigma^* ab$ $R_2 = ba\Sigma^*$, $R_1 \circ R_2 = \Sigma^* abba \Sigma^*$

$\Sigma = \{a, b\}$



Input abbbabbbacbab...



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Part 2: Given a DFA/NFA ⇝ reg expression

e.g. DIV_BY_3 = {0, 3, 6, 9, 12, 15, ...}

DIV_BY_2 = {0, 2, 4, 6, 8, 10, 12, ...}

$$((0 \cup 2 \cup 4 \cup (6 \cup 8)) \cup ((1 \cup 2 \cup 3 \cup 4 \cup 5 \cup 6 \cup 7 \cup 8 \cup 9)\Sigma^*(0 \cup 2 \cup 4 \cup 6 \cup 8))$$

≤ 1 digit

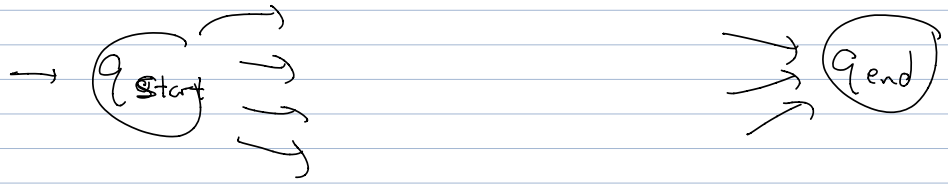
≥ 2 digits

could be ε ⇝ 2 digit number

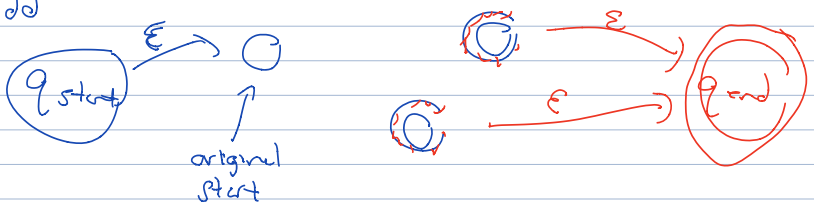
DIV_BY_2_LEADING_ZEROS_OK_EMPTY_STRING_NOT_OK

$\Sigma^*(0 \cup 2 \cup 4 \cup 6 \cup 8)$

There is a procedure: take NFA: insist!



Could add



Intermediate state