Homework:

All maps \( S \rightarrow T \)

All maps \( A^* \rightarrow \{\text{yes}, \text{no}\} \) decision problems

\( \Rightarrow \) language \( A \)

Section 1.2 Non-deterministic finite automata

1. This is important for \( P \) vs \( NP \)

2. We actually need to, has applications

\[ \{ \text{ruh, uh} \}^* = \{ \varepsilon, \text{ruh, uh, ruhuh, ...} \} \]

\[ \{ a^5, a^3 \}^* \cap \{ \text{abba, bcaab} \}^* \]

If \( L \) has "overlap," even knowing what \( L^* \) is can be tricky...

\[ \{ a^5, a^7 \}^* = \{ \varepsilon, a^5, a^7, a^{10} = a^5 a^5, a^{12} = a^5 a^7, ... \} \]

but \( a^{23} \in \uparrow \), but \( a^{24}, a^{25}, a^{26}, ... \) is
\[ \{a^5, a^7\}^* = \{ (a^5)^m (a^7)^n \mid m,n \in \mathbb{N} \} \]

DFA for \( \{a^5, a^7\} \) over \( \Sigma = \{a\} \):

\[ q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_2 \xrightarrow{a} q_3 \xrightarrow{a} q_4 \]

Use "non-determinism."

Called NFA, non-deterministic finite automaton

Rule: If input "abba," has at least one path to a final/accepting state, then input is accepted.

(1) If \( L \) is regular, then \( L^* \) is recognized by some NFA
(2) Any language recognized by an NFA, is recognized by some DFA
NFA for $\{a^5, a^3\}$ over $\Sigma = \{a\}$.

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NFA

\[ \begin{array}{c}
\qquad \text{NFA for } \{a^5, a^3\} \text{ over } \Sigma = \{a\}.

\begin{array}{c}
\text{\begin{array}{c}
\text{q}_0 \\
\text{q}_2 \\
\text{q}_4 \\
\text{q}_5 \\
\end{array}}
\end{array}
\end{array} \]
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Input $a^5$:
$q_0 \rightarrow q_1 \rightarrow q_2 \rightarrow q_3 \rightarrow q_4 \rightarrow q_5 \rightarrow q_0 \rightarrow \ldots$

We could change $q_5$ to $q_5$

Say $\{s \in \{a,b\}^* | s \text{ has } \textit{abba} \text{ or } \textit{bbba} \text{ as substring} \}$

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\begin{array}{c}
\text{Another example of where non-determinism makes building automaton easier.}

\begin{array}{c}
\text{Input } \textit{bbababaaabaaab}.
\end{array}
\end{array}
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Input: Which states could I possibly reach?

\[ \{q_0\} \]

\[ \{q_1\} \]

\[ \{q_2\} \]

\[ \{q_3, q_4\} \]

\[ \{q_7, q_0, q_2\} \]

In general: \( \text{Power}(Q) \): set of all subsets of states

Say \( \{q_0, q_3, q_2, q_1\} \) in an NFA, read \( c \) \( \{ \_ \_ \_ \_ \} \)
DFA recognizes \( L \)

recognize \( L \) \( \text{comp} = \emptyset \) \( L \)