Homework:

All maps $S \to T$

All maps $A^* \to \{yes, no\}$ decision problems $\Leftrightarrow$ languages over A

Section 1.2 Non-deterministic finite automata

① this is important for P vs NP

② We actually need to, has applications

$\{ruh, uh\}^* = \{\varepsilon, ruh, uh, ruhuh, \ldots\}$

$\{a^5, a^7\}^*$ $\{abba, baaab\}^*$

aaaaa aaaaaaa

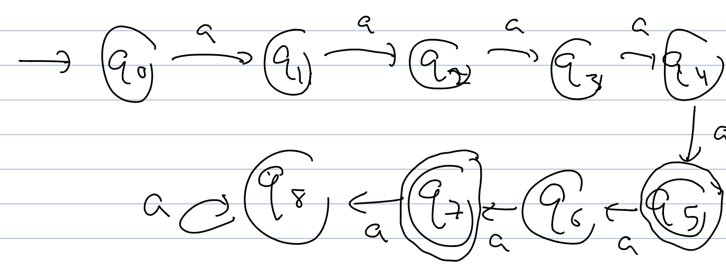
If L has "overlap," even knowing what L^* is can be tricky...

$\{a^5, a^7\}^* = \{\varepsilon, a^5, a^7, a^{10} = a^5 a^5, a^{12} = a^5 a^7, \ldots\}$

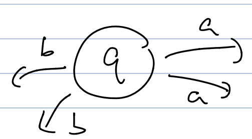
but $a^{23} \in$, but $a^{24}, a^{25}, a^{26}, \ldots$ is

$$\{a^5, a^7\}^* = \left\{ (a^5)^m (a^7)^n = a^{5m+7n}, \quad m,n = 0,1,2,\ldots \right\}$$

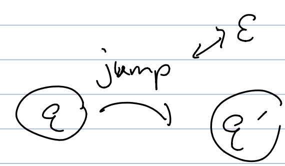
DFA for $\{a^5, a^7\}$ over $\Sigma = \{a\}$:

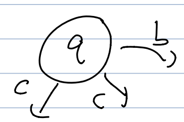


Use "non-determinism"



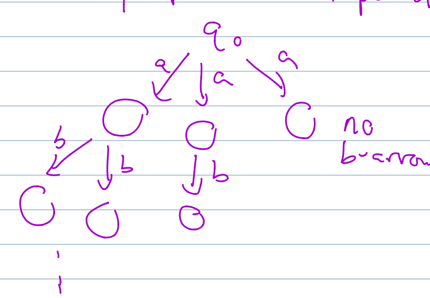
more than one choice





maybe no arrow leaving q labelled a

many paths input ab...



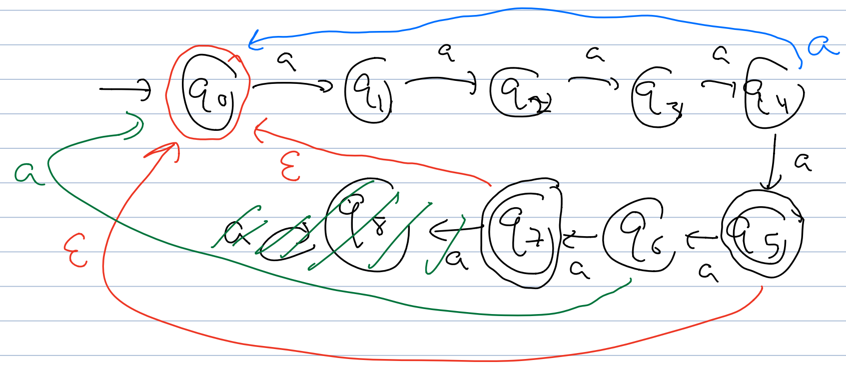
Called NFA non-deterministic finite automaton

Rule: If input "abba" has at least one path to a final/accepting state, then input is accepted.

(1) If L is regular, then L^* is recognized by some NFA

(2) Any language recognized by an NFA, is recognized by some DFA

NFA
~~DFA~~ for $\{a^5, a^7\}^*$ over $\Sigma = \{a\}$:



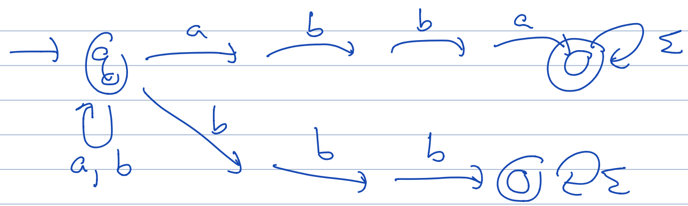
Alternate to q_5 jumps q_0

Alternative q_7 jump q_0

input a^{10} : $q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_2 \xrightarrow{a} q_3 \xrightarrow{a} q_4 \xrightarrow{a} q_5 \xrightarrow{\text{jump}} q_0 \xrightarrow{a} \ldots$

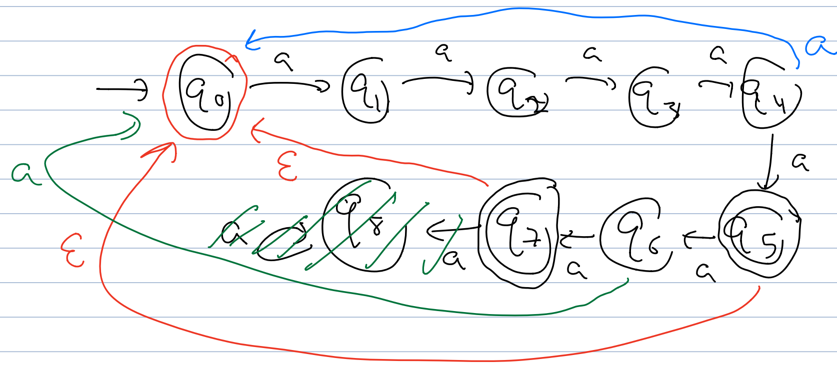
We could change q_5 to q_5

Say $\{ s \in \{a,b\}^* \mid s$ has abba or bbb as substring $\}$



Another example of where non-determinism makes building automaton easier

Input bbababbaaaaba...



Input — Which states could I possibly reach ?

$\{q_0\}$

a ———

a ——— $\{q_1\}$

a

$\{q_2\}$

a

after 5 a's

:

a $\{q_5, q_0\}$

a $\{q_6, q_1\}$

a $\{q_7, q_0, q_2\}$

=

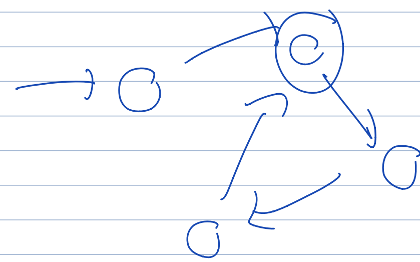
In general: Power(Q) = set of all subsets of states

say $\{q_0, q_3, q_{27}, q_{☺}\}$ in an NFA,

read c

$\{ \_ \; \_ \; \_ \}$

DFA recognizes L



recognize $L^{comp} = \Sigma^* \setminus L$