Remarks:

Very interesting

Section 1.2 Non-deterministic Finite Automata

Applications

Section 1.3 Regular Expressions

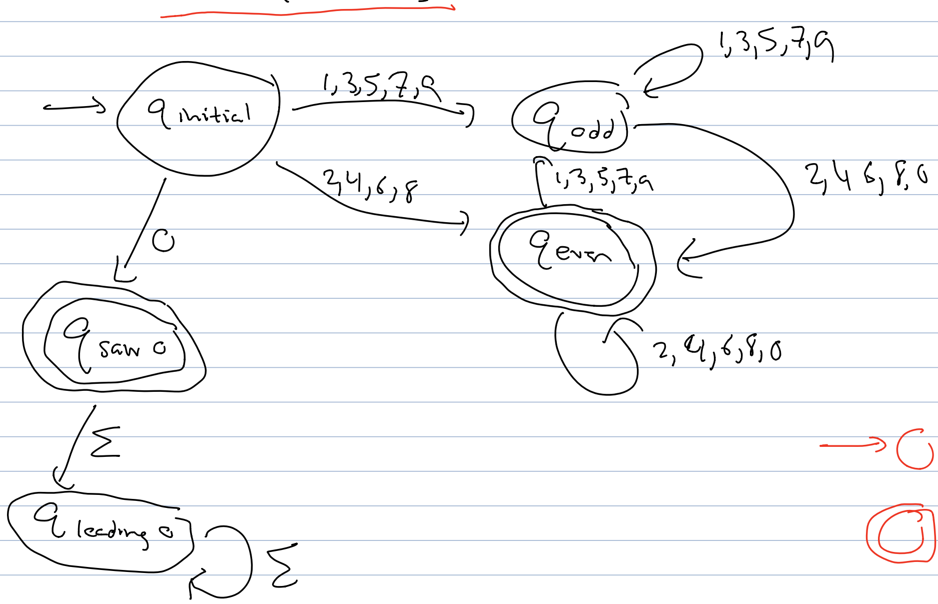
~~Section 1.4 Non-regular languages via the pumping lemma~~

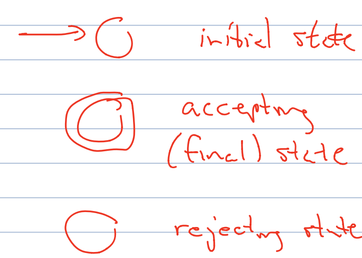
use Myhill-Nerode theorem

Last week:

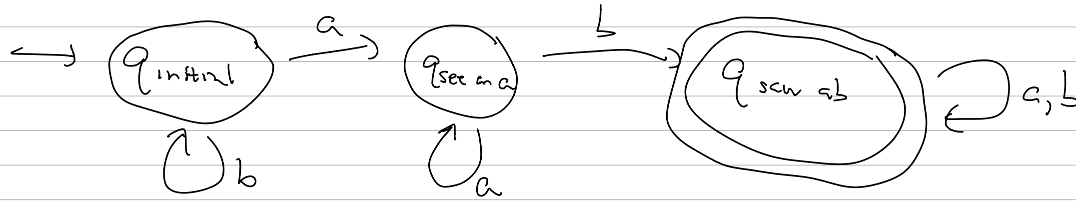
DIV_BY_2 = { 0, 2, 4, 6, 8, 10, 12, ... }

over $\Sigma = \{0, .., 9\}$





$\Sigma = \{a, b\}$, $\quad L = \{ s \mid ab \text{ is a substring of } s \}$



Say DIV_BY_2 has DFA L_1
DIV_BY_3 has DFA L_2

$\underbrace{\text{DIV\_BY\_6}}_{L_1 \cap L_2}$ ... ?

$= \{0, 6, 12, 18, \ldots\}$

transition function $Q \times \Sigma \longrightarrow Q$

L_1 DFA $= (Q, \Sigma, \delta, q_0, F) = M_1$

L_2 DFA $= (Q', \Sigma, \delta', q_0', F') = M_2$

Can we run M_1 and M_2 "in parallel" ?

<u>Method</u>:

input string $\sigma_1 \sigma_2 \sigma_3 \ldots$

M_1: $q_0 \to q \to q \to q$

etc.

M_2: $q_0' \to q' \to q' \to q'$

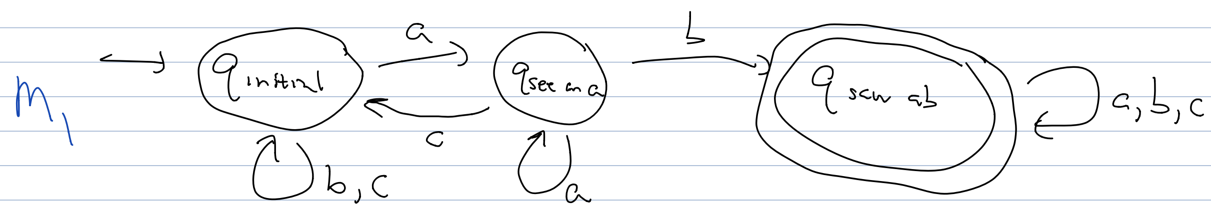
So "merge states"

$Q = \{ q_0, q_1, q_2, \ldots \}$

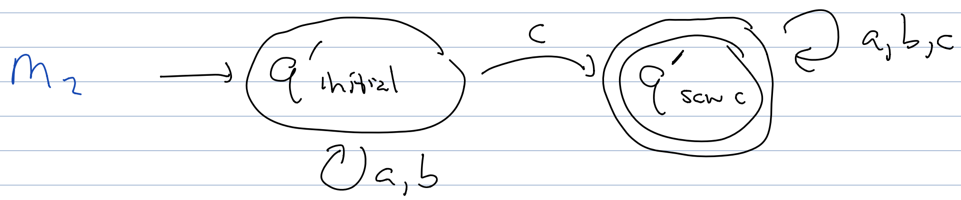
$Q' = \{ q_0', q_1', \ldots \}$

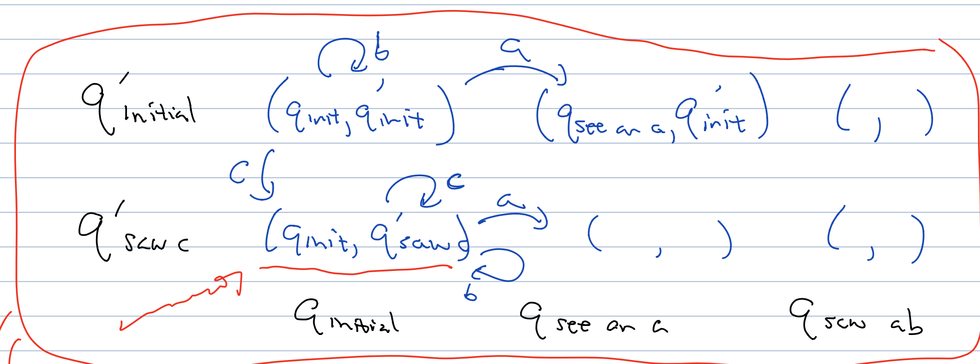
$Q \times Q' = \{ (q_0, q_0'), (q_0, q_1'), (q_1, q_0') \ldots \}$.

machine for ab is substring, $\Sigma = \{a, b, c\}$



machine is there at least one c





$Q' \times Q$

If $L_1 \cap L_2$ $\quad (q, q')$ is final if q is final for M_1,
q' " " " M_2

If $L_1 \cup L_2$ $\quad (q, q')$ is final if q is final OR q' is final in M_2

If $L_1 \setminus L_2$ or $L_2 \setminus L_1$

Here is what leads to Section 1.2 (non-determinism)

If L is a language over Σ,

$$L^* = \{ s_1 s_2 \cdots s_k \mid s_1, \ldots, s_k \in L \}$$

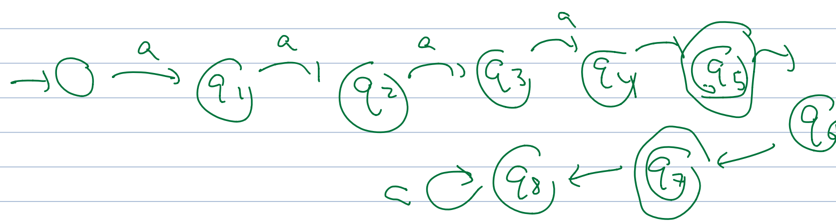
$$(\text{ruh}, \text{uh})^* = \{ \varepsilon, \text{ruh}, \text{uh}, \text{ruhuh}, \text{ruhruh}, \text{uhuh}, \ldots \}$$

$$\left(\substack{\text{words} \\ \text{language}} \right)^* = \_\_\_ \qquad (\text{how to parse})$$

If $\underbrace{L \text{ is regular}}_{\text{has a DFA}}$, then so is $\underbrace{L^*}_{\text{has a DFA}}$.

Example $\{a^5, a^7\}^*$ $\quad \Sigma = \{a\}$ $\quad (a^5 = aaaaa, \text{etc.})$

$\{a^5, a^7\}$ is regular



What is $\{a^5, a^7\}^* = \{ \varepsilon = a^0, a^5, a^7, a^{10}, a^{12},$

not a^8, a^{11}, a^{13}

$a^{22}, a^{23}, a^{24} \in \{a^5, a^7\}^* = ???$

$a^{23} \overset{???}{\in} \{a^5, a^7\}^*$ $\leftarrow$ non-trivial

$a^{24}, a^{25}, \ldots \in \{a^5, a^7\}^*$