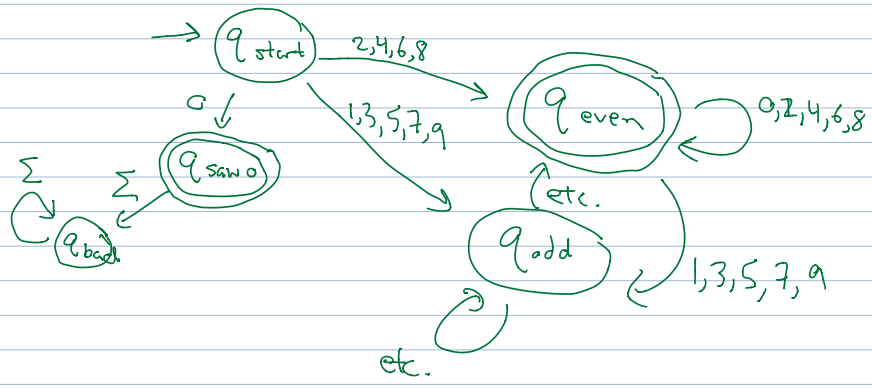
$\Sigma = \{0, 1, \ldots, 9\}$

DIV_BY_2 = $\{0, 2, 4, 6, 8, 10, 12, \ldots\}$

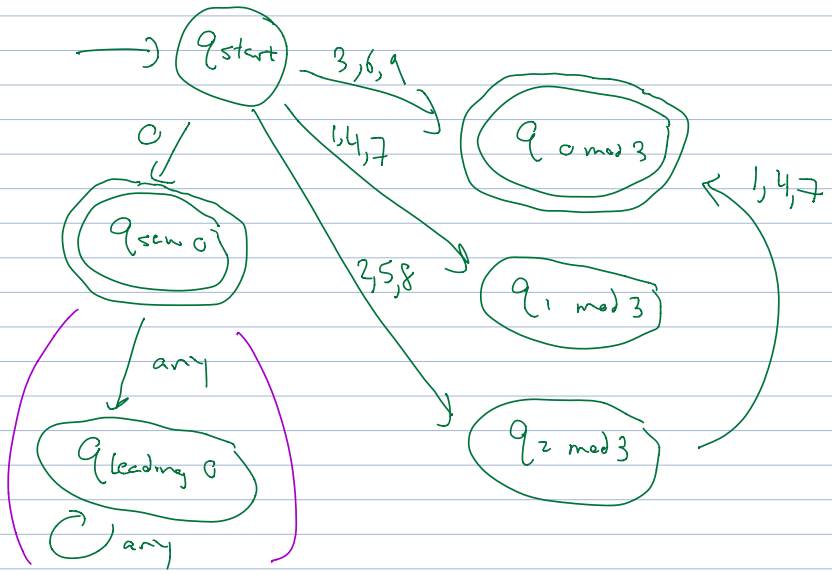


→ (q) start

(q) accepting (final)

(q) rejecting (not final)

DIV_BY_3 = $\{0, 3, 6, 9, 12, 15, \ldots\}$



DIV_BY_3 = $\{0, 3, 6, 9, 12, 15, \ldots\}$

Say $s_1 s_2 \ldots s_k$ say it begins | $s_2 \ldots s_k$

" " " " " | 2 $s_2 \ldots s_k$

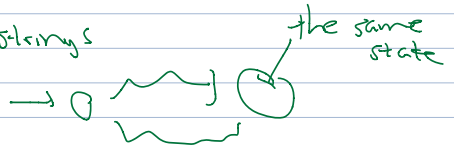
Start with 1 : 12, 150, 18, ... $\in$ DIV_BY_3

Look at $\{ s_2 \ldots s_k \mid 1 s_2 s_3 \ldots s_k \in \text{DIV\_BY\_3} \}$

$\{2,5,8,02,05,08,11,14,17,\ldots\}$ → add 1 in front → $\{12,15,18,102,105,108,111,\ldots\}$

Start with 2: you tack on $\{1,4,7,01,04,07,10,13,16,19,\ldots\}$ gives $\{21,24,27,201,204,207,210,213,\ldots\}$

Start with a 4: you can tack on $\{2,5,8,02,05,08,11,14,17,\ldots\}$



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Given a language L over Σ, let s first, then s'

$$\text{AcceptableFuture}_L(s) = \{ s' \in \Sigma^* \mid ss' \in L \}$$

(1) $\text{AccFuture}_{\text{DIV\_BY\_3}}(1) = \{2,5,8,02,05,08,11,14,\ldots\}$

(2) $\text{AccFut}_{\text{DIV\_BY\_3}}(2) = \{1,4,7,01,04,07,10,13,\ldots\}$

(3) $\text{AccFut}_{\text{DIV\_BY\_3}}(02175) = \emptyset$ (leading 0 ⇒ ☹)

(4) $\text{AccFut}_{\text{DIV\_BY\_3}}(3693) = \{\varepsilon, 0,3,6,00,03,09,12,15,\ldots\}$

(5) $\text{AccFut}_{\text{DIV\_BY\_3}}(0) = \{\varepsilon\}$

(6) $\text{AccFut}_{\text{DIV\_BY\_3}}(\varepsilon) = \text{DIV\_BY\_3} = \{0,3,9,12,15,\ldots\}$

Names: ToAccept, 2Accept, 2ok, TackOn

But ① — ⑥ have to be in different states...

=

Is $L = \{01, 0011, 000111, 0^4 1^4, 0^5 1^5, \dots\}$

$(0^4 = 0000)$ $= \{0^n 1^n \mid n \in \mathbb{N}\}$ regular?

Intuition: if see $\underbrace{000000\dots00}$ need to remember how many

$TackOn(0) = \{1, 011, 0011, \dots\} = \{0^{n-1} 1^n \mid n \in \mathbb{N}\}$

$TackOn(00) = \{11, 0111, 001111, \dots\}$

$\vdots$ $(000) = \{111, 01^4, \dots\}$

$\vdots$

$TackOn(0^k) = \{1^k, 01^{k+1}, \dots\}$

} Infinitely many TackOn's

$\Rightarrow$ L is not regular