Last time:

Textbook (Sip) Ch 1 ~~~ machine ... algorithm

Ch 2 ~~~

Ch 3

Ch 4 ~~~

Section 4.2

Section 1 — (1) Halting can't be decided

related

Section 2 — (2) "Diagonalization method"

$\mathbb{Z}$ integers } countable

$\mathbb{Q}$

"Self-referencing"

"Negation" $\mathbb{R}$ } uncountable

Section 1: ① - "I am lying"

② - "This statement is false" ← statement s:

s is true $\Rightarrow$ s is false

s is false $\Rightarrow$ s is not false $\overset{?}{\Rightarrow}$ s is true.

Thm: If you can build such statement, S, then either: - S is true and s is false

- s is neither true nor false

③

Seems maybe ok { Leslie writes about all (and only about) those who do <u>not</u> write about themselves.

Q: Does Leslie write about Leslie?

If Leslie writes about Leslie $\Rightarrow$ Leslie does not write about Leslie

" " does not write " " $\Rightarrow$ Leslie writes " "

④ Let x be the smallest positive integer not described by a phrase of fewer than fifty words.

What is x ?

Say $\underbrace{x = 12{,}121{,}121{,}121{,}121}$. Then

x is described as

"the smallest positive integer ---"

① – ④ are "paradoxes"

⑤ – ⑦ includes informal proof that the Halting problem can't be solved

$\mathbb{R}$ are uncountable

Cantor's thm

{ strings over alphabet, A } = A^* is countable, but

{ languages over alphabet, A } is uncountable

$\mathbb{R}$ are uncountable:

Say you have a map $f: \mathbb{N} \to \mathbb{R}$, i.e.

$r_1 = f(1), \quad r_2 = f(2), \quad r_3 = f(3), \ldots$

Want to find $r \in \mathbb{R}$ s.t. $r \neq r_1, r \neq r_2, r \neq r_3, \ldots$

digit 1 digit 2

(idea) say:

$r_1 = 37.021356\ldots$

$r_2 = 0.000329\ldots$

$r_3 = 12.12345\ldots$

$r_4 = .311111$

(pick either 1 or 2)

$r = 0.1112\ldots$

$r \neq r_1$ by digit #1

$r \neq r_2$ " " #2

⋮

[Sup]:

$.1350000\ldots$

$= .1349999\ldots$

Question: Can we prove $\mathbb{Q}^+$ are not countable?

$r_1 = 0.33333 =$

$r_2 = 0.1428\ldots$

$r_3 = 0.2520\ldots$

$r_4 = 0.1111111$

$r = 0.1112\ldots$

no guarantee that r is rational

Is $\mathbb{N}$ countable?

$$\begin{aligned} 1 &= n_1 \\ 2 &= n_2 \\ 7154 &= n_3 \\ &\vdots \end{aligned}$$

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