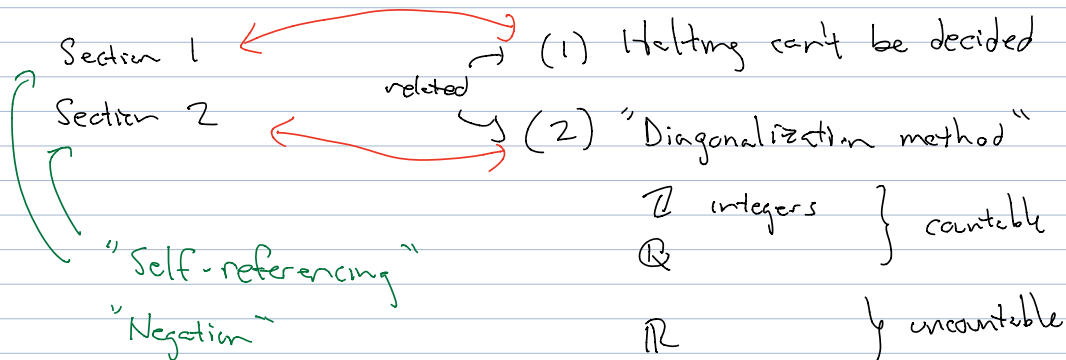


Last time:

Textbook (Sip) Ch 1 - - - machine - - - algorithm
Ch 2 - - -
Ch 3
Ch 4 . - - -

Section 4.2



Section 1: ① - "I am lying"

② - "This statement is false" \leftarrow statement s :

s is true \Rightarrow s is false

s is false \Rightarrow s is not false $\stackrel{?}{\Rightarrow}$ s is true.

Thm: If you can build such statement, s ,
then either: - s is true and s is false
- s is neither true nor false

③

Seems maybe OK { Leslie writes about ^{all} (and only about) those who do not write about themselves.

Q: Does Leslie write about Leslie?

If Leslie writes about Leslie \Rightarrow Leslie does not write about Leslie

" " does not write " " \Rightarrow Leslie writes " "

④ Let x be the smallest positive integer not described
by a phrase of fewer than fifty words.

What is x ? Say $x = 12, 121, 121, 121, 121$. Then
 x is described as

①-④ are "paradoxes"

"the smallest positive integer ---"

⑤-⑦ includes informal
proof that the Halting
problem can't be solved

\mathbb{R} are uncountable

Cantor's thm

{ strings over alphabet, A } = A^*
is countable, but
{ languages over alphabet, A } is
uncountable

\mathbb{R} are uncountable:

Say you have a map $f: \mathbb{N} \rightarrow \mathbb{R}$, i.e.

$$r_1 = f(1), r_2 = f(2), r_3 = f(3), \dots$$

Want to find $r \in \mathbb{R}$ s.t. $r \neq r_1, r \neq r_2, r \neq r_3, \dots$

(idea) say:

$r_1 =$	37	0	2	1	3	5	6	...
$r_2 =$	0.	0	0	0	3	2	9	...
$r_3 =$	12.	1	2	3	4	5	...	
$r_4 =$.	3	1	1	1	1	1	...

digit 1 → digit 2

$$r = 0.\underline{1}112 \dots$$

(pick either 1 or 2)

$r \neq r_1$ by digit #1

$r \neq r_2$ " " #2

⋮

[Snp]:

$$= 0.1350000 \dots$$

$$= 0.1349999 \dots$$

=

Question: Can we prove \mathbb{Q}^+ are not countable?

$$r_1 = 0.\underline{3}3333 \dots$$

$$r_2 = 0.\underline{1}428 \dots$$

$$r_3 = 0.25\underline{2}0 \dots$$

$$r_4 = 0.111111 \dots$$

$$r = 0.1112 \dots$$

no guarantee that
 r is rational

Is \mathbb{N} countable?

$$1 = n_1$$

$$2 = n_2$$

$$7154 = n_3$$

\vdots