

CPSC 421/501, Sept 6.

Short article on class website

Reading: Hendart on Uncountability, etc. } Topic 1

[Sip] Chapter 0 ← pick and choose

[Sip] Section 4.2, pages 202 (middle) to 207 (top)

out of nowhere → i.e. THE DIAGONALIZATION METHOD

Hendart

- Section 0 ~ Languages
- Section 1 ↔ Paradoxes, Thm (Halting Problem)
- Section 2 ↔ Cantor's Thm, Uncountability

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Bottom Line: How many algorithms? } Countable vs.

" " textbooks? }

" " PhD thesis }

The number of decision problems } Uncountable

" " " real numbers }

p 203 [Sip]: Injections, Surjections, Bijections, ...

5 Menus → 6 people

- (1) Even natural numbers { 2, 4, 6, 8, ... }
 - (2) Natural numbers $\mathbb{N} = \{ 1, 2, 3, 4, 5, \dots \}$
 - (3) Integers $\mathbb{Z} = \{ \dots -2, -1, 0, 1, 2, \dots \}$
 - (4) Reals \mathbb{R} } biases
- } Some size

Even naturals	2	4	6	8	10	...	same size (??)
Naturals	1	2	3	4	5	...	

Def S is countable if there is a surjective map $f: \mathbb{N} \rightarrow S$.

Another way: S is countable if either

- ① S is a finite set, OR
- ② There is a bijection $f: \mathbb{N} \rightarrow S$, i.e. correspondence

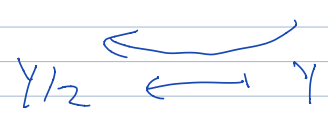
$$S = \{ f(1), f(2), f(3), \dots \}$$

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Injection, Surjection, Bijection, ...

And: $\mathbb{N} \rightarrow \{\text{even naturals}\}$

$$f(x) = 2x$$



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Injection: ID numbers, S.I.N.

$f: S \rightarrow T$ is injective if $s_1 \neq s_2 \in S, f(s_1) \neq f(s_2)$

e.g. $\{\text{UBC students}\} \rightarrow \{\text{8-digit numbers}\}$
 size 10^8

Surjection: 18 TA's, want give office hours every day of week

$f: S \rightarrow T$ is surjective (onto) if for each $t \in T$ there is some (at least one) $s \in S$ s.t. $f(s) = t$

Bijjective: - Both injective and surjective

- Or {perfect matching
one-to-one correspondence} $S \leftrightarrow T$

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S Countably infinite \Leftrightarrow there is a bijection $S \leftrightarrow \mathbb{N}$

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Example: $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$ are countable.

$$\mathbb{N} = \{1, 2, 3, \dots\}$$



$$f \downarrow \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & \dots \\ 0 & -1 & 1 & -2 & 2 & -3 & 3 & \dots \end{array}$$

$$f(x) = \begin{cases} (x-1)/2 & x \text{ odd} \\ -x/2 & x \text{ even} \end{cases}$$

Example: Positive Rational Numbers

$$\begin{array}{cccc} & & 1/1 & & \\ & & & & \downarrow \\ & & 2/1 & & 1/2 \\ & & & & & & & & \\ & & 3/1 & & \cancel{2/2} & & 1/3 & & \\ & & & & & & & & \\ 4/1 & & 3/2 & & 2/3 & & & & 1/4 \end{array}$$

	bijection		surjection	
1	1/1	}	1	1/1
2	2/1		2	2/1
3	1/2		3	1/2
4	3/1		4	3/1
5	2/2 1/3		5	2/2
	⋮			

Rem: If there is a surjection $S \rightarrow T$ and
 " " " " $T \rightarrow S$ then

there is a bijection $S \leftrightarrow T$

Example: \mathbb{R} is uncountable.

$$r_1 = 3.21791$$

$$r_2 = 4.31147$$

$$r_3 = -12$$

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