CPSC 421/501, Sept 6.

Short article on class website

Reading: Handout on Uncountability, etc. } Topic 1

[Sip] Chapter 0 ← pick and choose

[Sip] Section 4.2, pages 202 (middle) to 207 (top)

out of nowhere → i.e. THE DIAGONALIZATION METHOD

Handout
- Section 0 ~ Languages
- Section 1 ⟷ Paradoxes, Thm (Halting Problem)
- Section 2 ⟷ Cantor's Thm, Uncountability

Bottom Line: How many algorithms?
" " textbooks?
" " PhD thesis
} Countable

vs.

The number of decision problems
" " " real numbers
} Uncountable

p 203 [Sip]: Injections, Surjections, Bijections, ...

5 Menus ⟶ 6 people

(1) Even natural numbers $\{2, 4, 6, 8, \dots\}$

(2) Natural numbers $\mathbb{N} = \{1, 2, 3, 4, 5, \dots\}$

n

(3) Integers $\mathbb{Z} = \{\dots -2, -1, 0, 1, 2, \dots\}$

} Same size

(4) Reals $\mathbb{R}$ } bigger

Even naturals $\quad 2 \quad 4 \quad 6 \quad 8 \quad 10 \ \ldots$

$\qquad\qquad\qquad\quad \Updownarrow \quad \Updownarrow \quad \Updownarrow \quad \Updownarrow \quad \Updownarrow$ $\qquad$ same size ??

Naturals $\quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \ \ldots$

Def S is countable if there is a surjective map $f : \mathbb{N} \to S$.

Another way: S is countable if either

1. S is a finite set, OR
2. There is a bijection $f : \mathbb{N} \to S$, i.e.

correspondence

$$S = \{ f(1), f(2), f(3), \ldots \}$$

=

Injections, Surjections, Bijections, ...

And: $\mathbb{N} \to \{\text{even naturals}\}$

$$f(x) = 2x$$

$$y/2 \longleftarrow y$$

=

Injection: ID numbers, S.I.N.

$f : S \to T$ is injective if $s_1 \neq s_2 \in S$, $f(s_1) \neq f(s_2)$

e.g. $\{\text{UBC students}\} \longrightarrow \{\text{8-digit numbers}\}$

size 10^8

<u>Surjection</u>: 18 TA's, want give office hours every day of week

$f: S \to T$ is <u>surjective</u> (onto) if for each $t \in T$

there is some (at least one) $s \in S$ s.t. $f(s) = t$

<u>Bijective</u>: – Both <u>injective</u> and surjective

– Or {perfect matching / one-to-one correspondence} $S \leftrightarrow T$

S Countably infinite $\iff$ there is a bijection $S \leftrightarrow \mathbb{N}$

Example: $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ are countable.

$\mathbb{N} = \{1, 2, 3, \ldots\}$

$$\begin{array}{c|ccccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & \ldots \\ f \downarrow & 0 & -1 & 1 & -2 & 2 & -3 & 3 & \ldots \end{array}$$

$$f(x) = \begin{cases} (x-1)/2 & x \text{ odd} \\ -x/2 & x \text{ even} \end{cases}$$

Example: Positive Rational Numbers

$$\begin{array}{ccccccc} & & & 1/1 & & & \\ & & 2/1 & & 1/2 & & \\ & 3/1 & & \cancel{2/2} & & 1/3 & \\ 4/1 & & 3/2 & & 2/3 & & 1/4 \end{array}$$

bijection	
1	1/1
2	2/1
3	1/2
4	3/1
5	~~2/2~~ 1/3
⋮	

surjection	
1	1/1
2	2/1
3	1/2
4	3/1
5	2/2

Rem: If there is a surjection $S \to T$ and
" " " " $T \to S$ then

there is a bijection $S \Leftrightarrow T$

Example: $\mathbb{R}$ is uncountable.

$r_1 = 3.21791$

$r_2 = 4.31147$

$r_3 = -12$

⋮