

## HOMEWORK 7, CPSC 421/501, FALL 2019

JOEL FRIEDMAN

**Copyright:** Copyright Joel Friedman 2019. Not to be copied, used, or revised without explicit written permission from the copyright owner.

Please note:

- (1) You must justify all answers; no credit is given for a correct answer without justification.
- (2) Proofs should be written out formally.
- (3) Homework that is difficult to read may not be graded.
- (4) You may work together on homework, **but you must write up your own solutions individually**. You must acknowledge with whom you worked. You must also acknowledge any sources you have used beyond the textbook and two articles on the class website.

---

In these exercises, “the handout” refers to the article “Self-referencing, Uncountability, and Uncomputability” on the 421/501 homepage.

- (1) Let

$$L = \{\langle M \rangle \mid M \text{ is a T.m. that halts on input } \epsilon\}$$

(where  $\epsilon$  is the empty string). Show that  $L$  is (Turing) undecidable but (Turing) recognizable. What can you say about the complement of  $L$ ?

- (2) Let

$$L = \{\langle M \rangle \mid M \text{ is T.m. that uses all of its states}\}$$

(i.e., for each state,  $q$ , of  $M$ , there is some input to  $M$  on which  $M$  reaches  $q$ ). Show that  $L$  is (Turing) undecidable but (Turing) recognizable. What can you say about the complement of  $L$ ?

DEPARTMENT OF COMPUTER SCIENCE, UNIVERSITY OF BRITISH COLUMBIA, VANCOUVER, BC V6T 1Z4, CANADA.

*E-mail address:* [jf@cs.ubc.ca](mailto:jf@cs.ubc.ca)

*URL:* <http://www.cs.ubc.ca/~jff>

---

Research supported in part by an NSERC grant.