HOMEWORK 4, CPSC 421/501, FALL 2019

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Please note:

(1) You must justify all answers; no credit is given for a correct answer without justification.
(2) Proofs should be written out formally.
(3) Homework that is difficult to read may not be graded.
(4) You may work together on homework, **but you must write up your own solutions individually.** You must acknowledge with whom you worked. You must also acknowledge any sources you have used beyond the textbook and two articles on the class website.

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(1) Let DIV-BY-2 be the language

$$\{0, 2, 4, 6, 8, 10, 12, \ldots\}$$

over $\Sigma = \{0, 1, \ldots, 9\}$. In class we gave a 5 state DFA to recognize this language (see the beginning of the notes of Sept. 20). Take this DFA and convert this to a regular expression for DIV-BY-2 using the procedure described in [Sip] Section 1.3 (or see the notes for Sept. 30 and Oct. 2), as follows: (1) introduce a state q_{end} as a new unique final state, (2) then eliminate q_{bad}, then $q_{\mathrm{saw}\ 0}$, then q_{odd}, then q_{even}.

(2) For any string of length k, $s = \sigma_1\sigma_2 \ldots \sigma_k$ over an alphabet Σ, we define the *reverse string of* s to be the string

$$s^{\mathrm{rev}} = \sigma_k\sigma_{k-1} \cdots \sigma_2\sigma_1.$$

For any language, L over Σ, we define the *reverse of* L to be the language

$$L^{\mathrm{rev}} \stackrel{\mathrm{def}}{=} \{s^{\mathrm{rev}} \mid s \in L\}.$$

Research supported in part by an NSERC grant.

(a) Describe a general procedure that takes take a DFA or an NFA with n states that recognizes a language L, and produces an NFA recognizing $L^{\rm rev}$ with at most $n+1$ or $n+2$ states. [Hint: the set of states of the new NFA can be almost the same set of states.]
(b) Illustrate your construction on the DFA in Problem 1 for DIV-BY-2.
(c) If L is recognized by a DFA with n states, give an upper bound on the size of a DFA (i.e., number of states of a DFA) that recognizes $L^{\rm rev}$.

(3) Use the Myhill-Nerode theorem to show that the language

$$L = \{a^n b^m \mid n, m \in \mathbb{N}, n > m\}$$

over $\Sigma = \{a, b\}$ is not regular, by proving that the sets

$$\mathrm{AccFut}_L(a),\ \mathrm{AccFut}_L(a^2),\ \mathrm{AccFut}_L(a^3)\ , \ldots$$

are all different.

(4) Let $\Sigma = \{0, 1\}$.
(a) Show that the language $L = \Sigma^9 1 \Sigma^*$ is recognized by DFA with 12 states. [Hint: It is simplest to give a DFA and explain how it works.]
(b) Show that any DFA recognizing the language $L = \Sigma^* 1 \Sigma^9$ must have at least 1024 states. [Hint: One way to do this is to show that if s, s' are strings of length 10 over Σ, then $\mathrm{AccFut}_L(s) \neq \mathrm{AccFut}_L(s')$.]
(c) How does this relate to your answer in Problem 2(c)?

(End of Homework Problems to be Submitted for Credit.)

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