HOMEWORK 3, CPSC 421/501, FALL 2019

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Please note:

(1) You must justify all answers; no credit is given for a correct answer without justification.
(2) Proofs should be written out formally.
(3) Homework that is difficult to read may not be graded.
(4) You may work together on homework, **but you must write up your own solutions individually.** You must acknowledge with whom you worked. You must also acknowledge any sources you have used beyond the textbook and two articles on the class website.

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(1) Let L be the language over $\Sigma = \{0, 1, \ldots, 9\}$ of integers divisible by 3, where we allow leading 0's and we consider the empty string, ϵ, to lie in L; hence

$$L = \{\epsilon, 0, 3, 6, 9, 00, 03, 06, 09, 12, 15, 18, 21, \ldots\}.$$

(a) Build a DFA with 3 states that recognizes L.
(b) Describe the set $\mathrm{AccFut}_L(s)$ for these values of s

$$\epsilon, 0, 1, 2, 3$$

(either describe the set completely or list those strings of length 0 and 1 in the set, and a few of length 2). Which of these are different, and which are the same? Justify your answer.
(c) Prove, using the Myhill-Nerode theorem, that any DFA accepting L must have at least 3 states.

(2) Let L be the language over $\Sigma = \{0, 1, \ldots, 9\}$ of integers divisible by 4, where we allow leading 0's and we consider the empty string, ϵ, to lie in L; hence

$$L = \{\epsilon, 0, 4, 8, 00, 04, 08, 12, 16, 20, 24, \ldots\}.$$

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(a) Describe the set $\mathrm{AccFut}_L(s)$ for these values of s

$$\epsilon, 0, 1, 2, 3$$

(either describe the set completely or list those strings of length 0 and 1 in the set, and a few of length 2). Which of these are different, and which are the same? Justify your answer.

(b) Prove, using the Myhill-Nerode theorem, that any DFA accepting L must have at least 3 states.

(c) Build a DFA with 3 states that recognizes L.

(3) Let L be the language of strings over $\Sigma = \{0, 1\}$ that are of length at least three and whose third symbol is a 1, i.e.,

$$L = \Big\{ \sigma_1 \dots \sigma_n \;\Big|\; \sigma_1, \dots, \sigma_n \in \Sigma,\ n \geq 3,\ \sigma_3 = 1 \Big\}.$$

(a) Build a DFA with 5 states that recognizes L.

(b) Use the Myhill-Nerode theorem to show that any DFA that recognizes L must have 5 states. [Hint: Consider $\mathrm{AccFut}_L(s)$ for the following 5 values of s: $\epsilon, 0, 00, 000, 001$, and show that these sets are all different.]

(4) Let L be the language of strings over $\Sigma = \{0, 1\}$ that are of length at least three and whose third last symbol is a 1, i.e.,

$$L = \Big\{ \sigma_1 \dots \sigma_n \;\Big|\; \sigma_1, \dots, \sigma_n \in \Sigma,\ n \geq 3,\ \sigma_{n-2} = 1 \Big\}.$$

(a) Write an NFA that recognizes L and has at most 4 states, and explain how your NFA works.

(b) Describe the sets $\mathrm{AccFut}_L(s)$ for all s of length three, i.e., for the 8 values of s

$$000, 001, 010, 011, 100, 101, 110, 111$$

(either describe the set completely or list those strings of length 0, 1, and 2 in the set). Are any of these two sets the same? Explain.

(c) Prove that any DFA recognizing L has at least 8 states.

(End of Homework Problems to be Submitted for Credit.)

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