

## A REGULAR EXPRESSION FOR DIV-BY-3

JOEL FRIEDMAN

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In class we derived a regular expression for DIV-BY-3, defined as

$$L = \{0, 3, 6, 9, 12, 15, \dots\}$$

(which is the language over  $\Sigma = \{0, 1, \dots, 9\}$ , and we do not allow leading zeros in elements of  $L$  and we do not consider the empty string to be part of  $L$ ).

In class of October 2, 2019 (see class notes), began with a five state NFA, added an ending state, and then eliminated the intermediate states. The expression we got was

$$0 \cup T_0 T_1^*,$$

where

$$T_0 = S_2 \cup S_3 S_4^* S_5, \quad T_1 = S_0 \cup S_1 S_4^* S_5,$$

where

$$S_0 = R_0 \cup R_2 R_0^* R_1, \quad S_1 = S_3 = R_1 \cup R_2 R_0^* R_2, \quad S_2 = R'_0 \cup R_2 R_0^* R_1,$$

$$S_4 = R_0 \cup R_1 R_0^* R_2, \quad S_5 = R_2 \cup R_1 R_0^* R_1,$$

where

$$R'_0 = 3 \cup 6 \cup 9, \quad R_0 = 0 \cup 3 \cup 6 \cup 9, \quad R_1 = 1 \cup 4 \cup 7, \quad R_2 = 2 \cup 5 \cup 8.$$

In the above expressions we have omitted some necessary parentheses.

Repeated substitution yields the following regular expression (after adding needed parentheses):

$$0 \cup \left( \left( (3 \cup 6 \cup 9) \cup (2 \cup 5 \cup 8)(0 \cup 3 \cup 6 \cup 9)^*(1 \cup 4 \cup 7) \right) \cup \left( (0 \cup 3 \cup 6 \cup 9) \cup (2 \cup 5 \cup 8)(0 \cup 3 \cup 6 \cup 9)^*(2 \cup 5 \cup 8) \right) \right)^* \left( \left( (0 \cup 3 \cup 6 \cup 9) \cup (1 \cup 4 \cup 7)(0 \cup 3 \cup 6 \cup 9)^*(2 \cup 5 \cup 8) \right) \left( (2 \cup 5 \cup 8) \cup (1 \cup 4 \cup 7)(0 \cup 3 \cup 6 \cup 9)^*(1 \cup 4 \cup 7) \right) \right) \left( \left( (0 \cup 3 \cup 6 \cup 9) \cup (2 \cup 5 \cup 8)(0 \cup 3 \cup 6 \cup 9)^*(1 \cup 4 \cup 7) \right) \cup \left( (0 \cup 3 \cup 6 \cup 9) \cup (2 \cup 5 \cup 8)(0 \cup 3 \cup 6 \cup 9)^*(2 \cup 5 \cup 8) \right) \right) \left( (0 \cup 3 \cup 6 \cup 9) \cup (1 \cup 4 \cup 7)(0 \cup 3 \cup 6 \cup 9)^*(2 \cup 5 \cup 8) \right)^* \left( (2 \cup 5 \cup 8) \cup (1 \cup 4 \cup 7)(0 \cup 3 \cup 6 \cup 9)^*(1 \cup 4 \cup 7) \right)^*$$

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(and we have added some colour to make things easier to check and navigate).

DEPARTMENT OF COMPUTER SCIENCE, UNIVERSITY OF BRITISH COLUMBIA, VANCOUVER, BC  
V6T 1Z4, CANADA.

*E-mail address:* `jf@cs.ubc.ca`

*URL:* `http://www.cs.ubc.ca/~jf`