ADDITIONAL MIDTERM PRACTICE, CPSC 421/501, FALL 2019

JOEL FRIEDMAN

All problems on Homework 1-6 are practice midterm problems.

(1) State which sets are countable, and which are uncountable; justify your answer[1].
   (a) $\mathbb{R}^2$.
   (b) $\mathbb{N} \times \mathbb{R}$.
   (c) $\mathbb{N} \times \mathbb{Z}$.
   (d) The set of all graphs.
   (e) The set of all graphs whose vertex equals $\{1, 2, \ldots, n\}$ for some integer n.
   (f) The set of all Turing machines.
   (g) The set of all Turing machines whose set of states equals $\{1, \ldots, q\}$ for some integer q, and whose tape alphabet equals $\{1, \ldots, \gamma\}$ for some integer γ.

(2) State whether the statements below are true or false, and justify your answer.
   (a) If L_1 and L_2 are regular languages over the same alphabet, then $L_1 \cup L_2$ is regular.
   (b) If L_1 and L_2 are regular languages over the same alphabet, then $L_1 \cap L_2$ is regular.
   (c) If L_1 and L_2 are regular languages over the same alphabet, then $L_1 \setminus L_2$ is regular.
   (d) If L is a regular language over Σ, then the complement of L in Σ^*, i.e., $\Sigma^* \setminus L$ is regular.
   (e) If L is a regular language, then L must be Turing decidable.
   (f) If L is a Turing decidable language, then L must be regular.

Research supported in part by an NSERC grant.

[1] In your justification, you may use the following facts:
(a) the following are equivalent conditions that S be countable: (1) S is finite or there is a bijection between $\mathbb{N}$ and S, (2) there is a surjection $\mathbb{N} \to S$, (3) there is an injection $S \to \mathbb{N}$.
(b) If $S \to T$ is surjective and S is countable, then T is countable,
(c) If $S \to T$ is injective and S is uncountable, then T is uncountable.
(d) For any set S, there is no surjection from $S \to \text{Power}(S)$.
(e) The $\mathbb{R}$ is uncountable.
(f) For any alphabet A, A^* is countably infinite.

(3) For each of the following languages, describe a DFA recognizing these languages; you may either draw a diagram using the notation in class and [Sip], or explicitly give $Q, \Sigma, \delta, q_{\rm init}, F$. Explain how your DFA works.

(a)
$$L = \{s \in \{a,b\}^* \mid |s| \bmod 3 \text{ eqauls } 2\} = ss(sss)^*$$
($|s|$ refers to the length of s).

(b)
$$L = \{s \in \{a,b\}^* \mid s \text{ ends with } aba\} = \{a,b\}^* aba$$

(4) For each of the languages in Problem 3, describe a 1-tape Turming machine that decides these languages. (Including an explicit description of $\Sigma, \Gamma, q_{\rm acc}, q_{\rm rej}$ and—either with a diagram or lists—$\Gamma, Q, q_{\rm init}$.) Explain how your Turing machine works.

(5) Describe an NFA (using a diagram or a list of Q, all δ values, F, $q_{\rm init}$) for the langauge $(a^5, a^7)^*$ (over the alphabet $\Sigma = \{a\}$) that uses 10 states or fewer. Explain how your NFA works.

(6) A language L over the alphabet $\Sigma = \{a\}$ has a^3 as its longest word. Use the Myhill-Nerode theorem to prove that any DFA recognizing L must have 5 states.

(7) Consider a map from $\mathbb{N} \to \text{Power}(\mathbb{N})$ such that $1 \notin f(1)$. Explain why $f(1)$ cannot equal
$$T = \{n \in \mathbb{N} \mid n \notin f(n)\}.$$

Department of Computer Science, University of British Columbia, Vancouver, BC V6T 1Z4, CANADA.
E-mail address: jf@cs.ubc.ca
URL: http://www.cs.ubc.ca/~jf