1. Circle either T for true, or F for false, for each of the statements below:

The intersection of a regular language and a regular language is regular. T F

The intersection of a regular language and a nonregular language is nonregular. T F

If $L$ is nonregular, then $L^*$ is nonregular. T F

If $L_1$ is regular and $L_1 \cap L_2$ is nonregular, then $L_2$ is nonregular T F

If $L$ is regular then $L$ is Turing decidable. T F
2. Give a Turing machine that takes as input, $x \in \{0, 1\}^*$, and (1) accepts $x$ if the length of $x$ is divisible by 3, and (2) rejects $x$ otherwise. You must explain how your machine works, and explicitly write your choice of $Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject}$. To describe $\delta$, you may (1) list its values, or (2) use a diagram as used in Sipser's textbook (and class).
3. Let $L = \{1^3, 1^5\}$. Recall that $L^* = \{\epsilon, 1^3, 1^5, 1^6, 1^8, 1^9, 1^{10}, \ldots\}$.

(a) [10 points] Prove that any DFA that recognizes $L$ must have six states, using the pumping lemma.

(b) [10 points] Prove that any DFA that recognizes $L^*$ (note the star) must have eight states, using the Myhill-Nerode theorem.

(c) [5 points] Draw an NFA state diagram with five states that recognizes $L^*$ and explains how your NFA works.
The University of British Columbia
Midterm Examinations - November 2017

Computer Science 421/501

Closed book examination Time: 45 minutes

Name __________________________ Signature __________________________
Student Number ________________ Instructor’s Name ________________
Section Number ________________

Special Instructions:
You may bring one two-sided 8.5 x 11 sheet of notes. No aids may not be used. Answer questions on the exam. This exam is two-sided!

Rules governing examinations

1. Each candidate should be prepared to produce his library/AMS card upon request.
2. Read and observe the following rules:
   No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
   CAUTION - Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
   (a) Making use of any books, papers or memoranda, other than those authorized by the examiners.
   (b) Speaking or communicating with other candidates.
   (c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.
3. Smoking is not permitted during examinations.

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