

Marks

- [10] 1. Describe a Turing machine that takes as input,  $x \in \{a, b\}^*$ , and (1) accepts  $x$  if  $|x|$  is even, and (2) rejects  $x$  if  $|x|$  is odd. You should **explicitly write** and **explain** each of  $Q, \Gamma, q_0, q_{\text{accept}}, q_{\text{reject}}, \delta$ .

- [10] **2.** Let 4SAT be the language of 4cnf's (conjunctions of disjunctions of 4 literals). Give a direct polynomial time reduction to show that  $3SAT \leq_P 4SAT$ .

- [10] **3.** Let  $L_{\text{agree}}$  be (as in class) the language of  $\langle M, N \rangle$  such that  $M$  and  $N$  are Turing machines that give the same result (accept, reject, or loops) on all inputs. Show that  $L_{\text{yes}} \leq L_{\text{agree}}$ .

[ Note:  $L_{\text{yes}}$  is the language of encodings of pairs  $M, x$  where  $M$  accepts  $x$ .]

- [10] 4. Recall how we showed  $L_{\text{yes}}$  is undecidable. Assume to the contrary that there is a program,  $P$ , that decides  $L_{\text{yes}}$ . Let  $D$  be a program such that for all programs,  $Q$ ,

$$\text{Result}(D, \text{EncodeProg}(Q)) = \neg \text{Result}(P, \text{EncodeBoth}(Q, \text{EncodeProg}(Q)))$$

Argue that considering the value of  $\text{Result}(D, \text{EncodeProg}(D))$  leads to a contradiction.

**The End**

Be sure that this examination has 6 pages including this cover

The University of British Columbia

Midterm Examinations - November 2009

Computer Science 421/501

Closed book examination

Time: 50 minutes

Name \_\_\_\_\_ Signature \_\_\_\_\_

Student Number \_\_\_\_\_ Instructor's Name \_\_\_\_\_

Section Number \_\_\_\_\_

### Special Instructions:

Calculators, notes, or other aids may not be used. Answer questions on the exam. This exam is two-sided!

### Rules governing examinations

**1. Each candidate should be prepared to produce his library/AMS card upon request.**

**2. Read and observe the following rules:**

No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.

Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

CAUTION - Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

(a) Making use of any books, papers or memoranda, other than those authorized by the examiners.

(b) Speaking or communicating with other candidates.

(c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

**3. Smoking is not permitted during examinations.**

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2		10
3		10
4		10
Total		40