CPSC 303 Feb 7, 2024

- Recurrence Relations and Finite Precision (Section 5)
- Normal & Subnormal Numbers in Double Precision

Rem: [A&G] only covers "normal numbers"

Next Up: Chapter 10 [A&G] or Interpolation (Polynomial)

Say

$$(\sigma - 1)(\sigma - s)(x_n) = 0$$

$$\left(\sigma^2 - (1+s) + s\right)(x_n) = 0$$

s likely; $0 < s < 1$

General solution

$$x_n = c_1 \cdot 1^n + c_2 s^n$$

$$= c_1 + c_2 s^n$$

Claim: This can have unexpected results

So for any c_1, $\quad 0 < |s| < 1$

$$\lim_{n\to\infty} \left(c_1 + c_2 s^n\right)$$

$$= c_1 + c_2 \lim_{n\to\infty} s^n$$

$$= c_1$$

Strange behaviour:

$c_1 = 0$, $\quad c_2 = 1$, $\quad x_n = s^n$

$x_0 = s^0 = 1$, $\; x_1 = s^1 = s$,

$x_2 = s^2$, $\; x_3 = s^3$, ...

$s = \frac{1}{2},$

$$(\sigma - 1)(\sigma - \tfrac{1}{2})$$

$$= \sigma^2 - 3/2\, \sigma + \frac{1}{2}$$

$$(\sigma^2 - \tfrac{3}{2}\sigma + \tfrac{1}{2})(\{x_n\}) = 0$$

$$x_{n+2} - \frac{3}{2} x_{n+1} + \frac{1}{2} x_n = 0$$

$$x_{n+2} = \frac{3}{2} x_{n+1} - \frac{1}{2} x_n$$

Rem: For MATLAB,

$x \leftarrow$ array $\quad x \leftarrow$ cell

x_1 or $x\{1\} = 1$

x_2 or $x\{2\} = 1/2$

$\vdots$

$x_n = (1/2)^{n-1}$ in exact arithm.

$=$

Useful MATLAB

format long ← give more digits

format short ← gives fewer

Rem: Look at MATLAB's

x_n -- n large, cyclic, non-zero strange numbers

Say $F_0 =$ given, $F_1 =$ given

MATLAB $F_1 =$ given F_2 given

$$F_{n+2} = F_{n+1} + F_n$$

If $F_1 = 1$, $F_2 = \dfrac{1-\sqrt{5}}{2}$

then get

$$F_n = \Bigg(\underbrace{\frac{1-\sqrt{5}}{2}}_{-0.6180\ldots} \Bigg)^{n-1}$$

similarly ...

$$f_n / (f_2)^{n-1} = 1$$

Double presision!

A real double precision number is

$$\pm 1.b_1 \dots b_{52} \times 2^m$$

in binary scientific: $\_ . \_ \_ \_ \times 2^{35}$

has to be 1

$$-1022 \leq m \leq 1023$$

So $m = -1022, -1021, \ldots, 1023$

possible values is

$$(1023) - (-1022) + 1$$

$$= 2046$$

There's 2 values left if

m is to have 11 bits

$2^{11} = 2048$

$\pm \}$ 1 bit

$b_1 \sim b_{52} \}$ 52 bits

$m \}$ at most 2048 values

11 bits

Total $= 1 + 52 + 11 = 64$

(One extra value of m is for

Inf, -Inf, NaN

infinity   Not a Number)

The other value of m

Note: in MATLAB:

2^{1023} is $8{,}988\_\_ \times 10^{307}$

2^{1024} is Inf

but

$(1/2)^{1074} = 4{,}94\ldots \times 10^{-324}$

$(1/2)^{1075} = 0$

In IEEE Double precision:

last value of m:

$$2^{-1022}$$

but no

$$2^{-1023}$$

It allow:

$$\pm 0.b_1 b_2 \ldots b_{52} \quad 2^{-1022}$$

Friday Start Interpolation