CPSC 303 Jan 8, 2024

"In mathematics you don't understand things. You just get used to them."

— John von Neumann

My alternate form:

"In mathematics it takes time for ideas and examples to sink in."

<u>Course website:</u>

https://www.cs.ubc.ca/~jf/courses/303.S2024/index.html

CPSC 303:

Parts of Ch 10-12, 14-16 of textbook by Ascher & Greif, (available online for free).

Topics:

Ch 10-13 Interpolation, Approximation

Ch 14-16 Differentiation, Integration, ODE's [PDE's]

Discussion: please post to piazza page.

If this fails, please email

To: jf@cs.ubc.ca

Subject: CPSC 303

Grading:

$$(10\%)\max(h,m,f) + (35\%)\max(m,f) + (55\%) f,$$

h = homework, m = midterm, f = final

Use: canvas.ubc.ca for
piazza and gradescope

Homework: Set Th 11:59 pm,
due Th 11:59 pm.

Individual Homework: You must write
up your own solution

Group Homework: You can make a
group submission, groups ≤ 4 people

Start: Intro to ODE's

§1.2, §4.2 (norms)

§14.2 diffentiation

§16.1, 2 ODE's

Heading towards

Ordinary Differential Equations

(Ch 16 [A & G])

Absolute vs. Relative Error!

if $v \in \mathbb{R}$ is an approx to

$u \in \mathbb{R}$, then

absolute error (in v) (as an

approximation to u) is

$$|u-v|$$

relative error $\dfrac{|u-v|}{|u|}$

$=$

The same works in $\mathbb{R}^n$:

$$\|\vec{u}\|_2 = \|(u_1, \dots, u_n)\|_2$$

$$= \sqrt{u_1^2 + u_2^2 + \dots + u_n^2}$$

Also use

$$\|\vec{u}\|_1 = |u_1| + \dots + |u_n|$$

$$\|\vec{u}\|_{max} = \|\vec{u}\|_\infty = \max_{1 \le i \le n} |u_i|$$

abs error in $\vec{v}$ as an approx

to $\vec{u}$ is $\|\vec{u} - \vec{v}\|_p$

rel error $$\frac{\|\vec{u} - \vec{v}\|_p}{\|\vec{u}\|_p}$$

where $p = 1, 2, \infty$.

Taylor's Theorem: (p.5)

$f:(a,b) \to \mathbb{R}, \quad (a<b,\ a,b \in \mathbb{R})$

$(a,b) \stackrel{\text{def}}{=} \left\{ x \in \mathbb{R} \mid a < x < b \right\}$

$f = f(x), \quad$ for $\quad a < x < b$

$f(x) \in \mathbb{R} = \{\text{real numbers}\}$

Assume f has $k+1$ derivatives

in (a,b):

$f(x)$ is f

$f'(x)$ is $\frac{df}{dx}$ or $f'(x)$

$f''(x)$ is $\left(f'(x)\right)' = f''$

$f'''(x) = f^{(3)}(x)$

$\vdots$

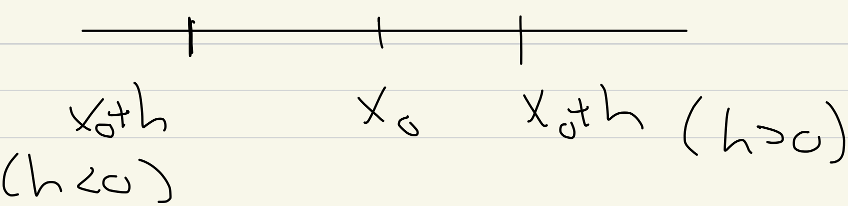
$f^{(k)}(x) = k^{\underline{th}}$ der of f at x

So $f^{(k+1)}(x)$ exists in (a, b)

We have $x_0, h \in \mathbb{R}$ s.t. [such that]

x_0, $x_0 + h$ lie in (a, b)

(remark h can be negative)



$$f(x_0+h) = f(x_0) + h f'(x_0)$$

$$+ \frac{h^2}{2} f''(x_0) + \dots + \frac{h^k}{k!} f^{(k)}(x_0)$$

$+$ error

$$\text{error} = \frac{h^{k+1}}{(k+1)!} f^{(k+1)}(\xi)$$

where ξ is between x_0 and x_0+h