INDIVIDUAL HOMEWORK 9, CPSC 303, FALL 2024

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Please note:

(1) You must justify all answers; no credit is given for a correct answer without justification.
(2) Proofs should be written out formally.
(3) Homework that is difficult to read may not be graded.
(4) You may work together on homework in groups of up to four, **but you must write up your own solutions individually and must acknowledge with whom you worked.** You must also acknowledge any sources you have used beyond the textbook and two articles on the class website.

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(1) Consider the ODE $y' = f(y)$ where $f(y) = 4y + 5$.
  (a) Describe the recurrence given by Euler's method with step size $h > 0$ for solving this ODE.
  (b) Describe the recurrence given by the Explicit Trapezoidal method (page 494 of [A&G]) with step size $h > 0$ for solving this ODE.
  (c) Find the exact solution to the above ODE subject to $y(0) = 1$, and for small $h > 0$ show that

$$y(h) = 1 + 9h + 18h^2 + 24h^3 + O(h^4)$$

  [Hint: for $|\delta|$ small, we have $e^\delta = 1 + \delta + \delta^2/2 + \delta^3/3! + O(\delta^4)$.]
  (d) Show that for Euler's method, the difference between y_1 and the true value of $y(h)$ is $O(h^2)$.
  (e) Show that for the Explicit Trapezoidal method, the difference between y_1 and the true value of $y(h)$ is $O(h^3)$.
  (f) Say that $a, b \in \mathbb{R}$ with $a \neq 1$. Find the general solution to the recurrence equation $y_{n+1} = ay_n + b$.
  (g) Use the last part to solve the recurrence given in Euler's method, and solve it for the initial condition $y_0 = 1$.

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(h) Let $m \in \mathbb{N}$ and let $h = 1/m$ in Euler's method; compare the exact value of $y(2)$ of the above ODE subject to $y(0) = 1$ with the Euler's method recurrence value of y_{2m} subject to $y_0 = 1$. Show that they differ to within $O(1/m)$ as $m \to \infty$.

(i) (This part won't be graded; so no need to hand it in.) Do parts (g,h) with Euler's method replaced by the Explicit Trapezoidal method, and show the numerical approximation and the exact answer differ by $O(1/m^2)$ (you might also check that the difference is not as small as $O(1/m^3)$, so $O(1/m^2)$ is the "true order of the difference").

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