EXTRA PRACTICE 5, CPSC 303, SPRING 2024

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Please note:

(1) You must justify all answers; no credit is given for a correct answer without justification.
(2) Proofs should be written out formally.

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(1) Find the general solution to the recurrence equation

$$x_{n+2} - 7x_{n+1} + 12x_n = n + 5.$$

(2) Find the general solution to the ODE

$$y'' - 7y' + 12y = t + 3.$$

(3) Find the general solution to the recurrence equation

$$x_{n+2} - 6x_{n+1} + 9x_n = n + 5.$$

(4) Find the general solution to the ODE

$$y'' - 6y' + 9y = t + 3.$$

(5) What is the general solution to the recurrence equation

$$(\sigma - 4)(\sigma - 5)^2(x_n) = 0?$$

Write down a formula, and show that for any $x_0, x_1, x_2 \in \mathbb{R}$, you can find the (unique) solution to this recurrence.

(6) Let $p(t)$ is any polynomial of degree $d \geq 1$, i.e., of the form $c_d t^d + c_{d-1} t^{d-1} + \cdots + c_0$ with $c_d \neq 0$ and $d \geq 1$. Then if $y(t) = p(t)e^{3t}$ we have

$$\left(\frac{d}{dt} - 3\right) y(t) = q(t)e^{3t}$$

where $q(t)$ is of degree $d - 1$.

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(7) In class we've seen that $(1/2)^{1075}$ is reported by MATLAB as 0, and $(1/2)^{1074}$ as $4.9406\ldots \times 10^{-324}$.
   (a) If you type `(3/2)*(1/2)^1074` into MATLAB, can MATLAB tell you that this number is roughly $7.4109\ldots \times 10^{-324}$ (since 1.5 times 4.9406 is roughly 7.4109)? Does double precision allow this? Explain this in terms of the subnormal numbers
   $$\pm 0.b_1 \ldots b_{52} \times 2^{-1022}$$
   using the fact that $(3/2) = 1 + (1/2)$, and therefore is represented as 1.1 in binary.
   (b) If you type `(3/2)*(1/2)^1073`, will double precision be able to record this number exactly? Explain, based on what we know about subnormal numbers.
   (c) What does MATLAB report for the value of `(3/2)*(1/2)^(1073)*2^1073` into MATLAB, will you get `1.5000...` ? Explain in terms what we know about double precision.
   (d) What does MATLAB report for the value of `((3/2)*(1/2)^(1073)*2^1000)*2^73` into MATLAB? Explain in terms what we know about double precision.
   (e) If you type `(5/4)*(1/2)^1073`, will double precision be able to record this number exactly? Explain, based on what we know about subnormal numbers. [Hint: $5/4 = 1 + 0/2 + 1/4$, and hene $5/4 = 1.01$ in binary.]

(8) MORE PROBLEMS MAY BE ADDED.

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