FINAL PRACTICE 2, CPSC 303, SPRING 2024 (DOCUMENT IN PROGRESS)

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The following problems are sample final exam problems, largely based on the homework. There are a lot on material after the midterm, i.e., starting with divided differences, but some cover earlier material.

(1) Show that for any there is a unique polynomial $p(x)$ of degree at most 3 such that $p(1), p'(1), p(2), p'(2)$ are any four given values.

(2) Show that there are unique reals c_0, c_1, c_2 such that for f sufficiently differentiable we have for fixed x and small h,

$$\frac{c_0 f(x+h) + c_1 f(x+2h) + c_2 f(x+3h)}{h^2} = f''(x) + O(h).$$

(3) Show that for each $n \geq 2$

$$U = \bigl[\sin(\pi/n) \quad \sin(2\pi/n) \quad \cdots \quad \sin\bigl((n-1)\pi/n\bigr)\bigr]^{\mathrm{T}}$$

is an eigenvector of $N_{\mathrm{rod},n-1}$. What is its eigenvalue?

(4) Consider the central force problem $m\ddot{x}(t) = -4mx(t)$.
  (a) What is the energy of the sytem?
  (b) If $x(0) = 1$, and $\dot{x}(0) = 4$, what is the energy of the system?
  (c) If $x(0) = 1$, and $\dot{x}(0) = 4$, show that for all t such that $x(t)$ exists, we have $x(t) \leq \sqrt{5}$.

(5) Let $a, y_0 \in \mathbb{R}$, and consider the explicit trapezoidal method applied to $y' = ay$ and $y(0) = y_0$ with step size h.
  (a) Show that this method leads to the recurrence

$$y_{i+1} = \bigl(1 + ah + (ah)^2/2\bigr) y_i$$

  where y_i is the approximation to $y(ih)$.
  (b) Explain why any second order method must satisfy

$$y_1 = \bigl(1 + ah + (ah)^2/2\bigr) y_0 + O(h^3)$$

(6) Consider the ODE
$$\mathbf{y}' = \begin{bmatrix} -300 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{y}$$
subject to $\mathbf{y}(0) = (2, 3)$.
   (a) What is the exact solution to $\mathbf{y}(t)$?
   (b) What approximate solution $\mathbf{y}_i$ (an approximation to $\mathbf{y}(ih)$) to this ODE given by Euler's method with step size $h = 1/100$?
   (c) In the approximate solution in part (b), is the first component of $\mathbf{y}_i$ strictly decreasing in i?
   (d) Answer parts (b) and (c) for backward Euler's method.

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