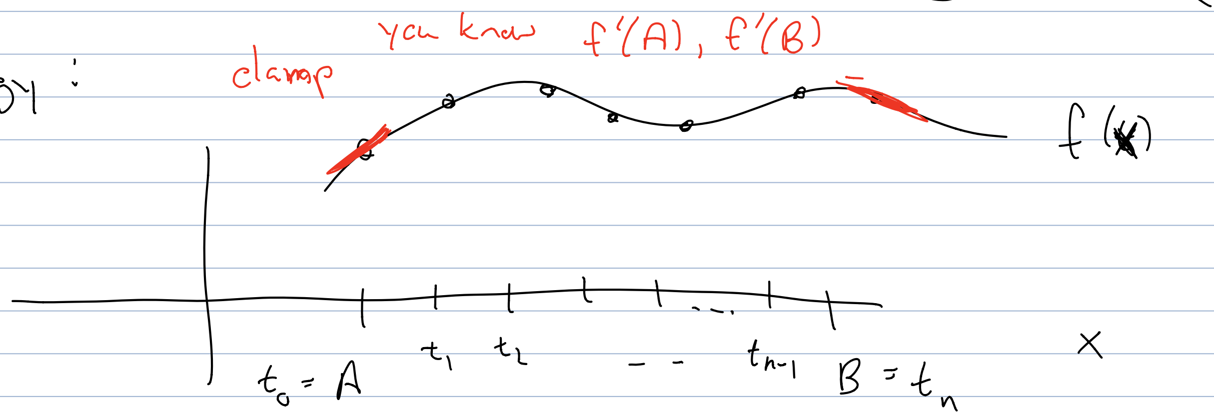
CPSC 303

Wed + $\frac{1}{2}$ Friday + $\frac{1}{2}$ Monday   11.1 — 11.3 Splines

$\frac{1}{2}$ Friday + $\frac{1}{2}$ Monday   Questions on HW + Midterm Problem Sets

Energy :



Look at

$$\mathcal{U} = \left\{ u : [A,B] \to \mathbb{R} \;\middle|\; \begin{array}{l} - \text{twice differentiable} \\ - u(t_i) = f(t_i) \quad i = 0, \dots, n \end{array} \right\}$$

Energy : $\mathcal{U} \to \mathbb{R}_{\geq 0}$: $$\mathcal{E}(u) = \int_A^B |u''(x)|^2 \, dx$$

Thm : Over $\mathcal{U}$, there is a unique $v \in \mathcal{U}$ where energy is minimized. Namely :

(1) v is a cubic spline with break points $t_1, \dots, t_{n-1}$,
i.e. v restricted to $[t_i, t_{i+1}]$ is a degree 3 poly

(2) and $v''(t_0)$, $v''(t_n) = 0$ "free conditions"

SPLINE

describes almost

SPLINE + $\underbrace{\text{"boundary conditions"}}_{2 \text{ conditions}}$

Boundary Conditions:

(1) free conditions:
$v''(A), v''(B) = 0$

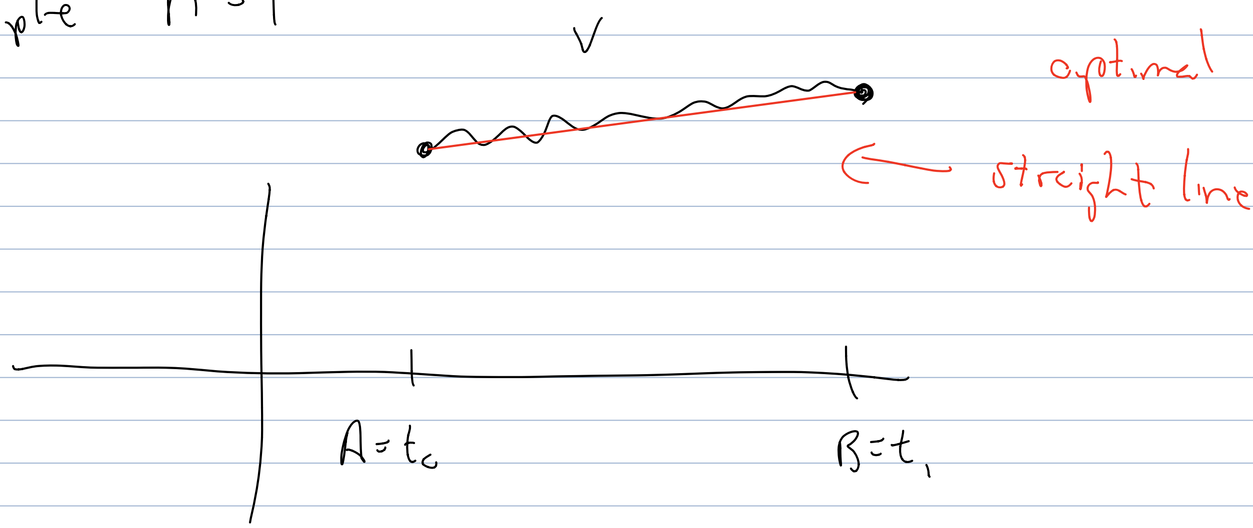
(2) clamped conditions
$v'(A), v'(B)$ given

(3) not-a-knot

in detail

(0) "initial conditions"
$v'(A), v''(A)$ given

Example $n = 1$



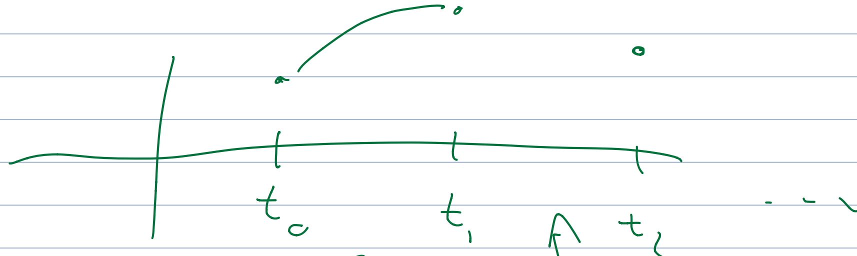
want $$\int_a^b |v''(x)|^2 \, dx = \mathcal{E}(v)$$

minimized.

If $v(x)$ = straight line, $v'' = 0$ ODE $\Rightarrow \mathcal{E}(v) = 0$

$v''(A), v''(B) = 0$ free

$n=2$

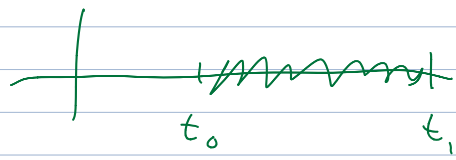


$S_0(x)$ deg ≤ 3 poly $\qquad$ $S_1(x)$ deg ≤ 3 poly

Notation (clever)

$$\underbrace{S_0(x) = a_0 + b_0(x-t_0) + c_0(x-t_0)^2 + d_0(x-t_0)^3}$$

ENERGY



$$S_0'(x) = b_0 + c_0\,2(x-t_0) + d_0\,3(x-t_0)^2$$

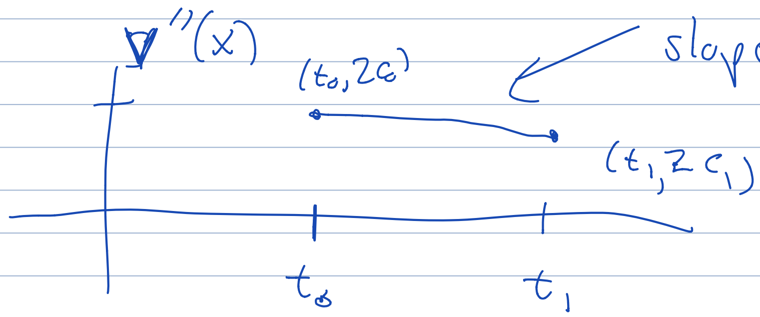
$$\boxed{S_0''(x) = 2c_0 + 6d_0(x-t_0)}$$

$$S_0(t_0) = a_0 \qquad S_0'(t_0) = b_0 \qquad S_0''(t_0) = 2c_0 \qquad S_0'''(t_0) = 6d_0$$

$$S_0''(t_1) = S_1''(t_1) = 2c_1$$

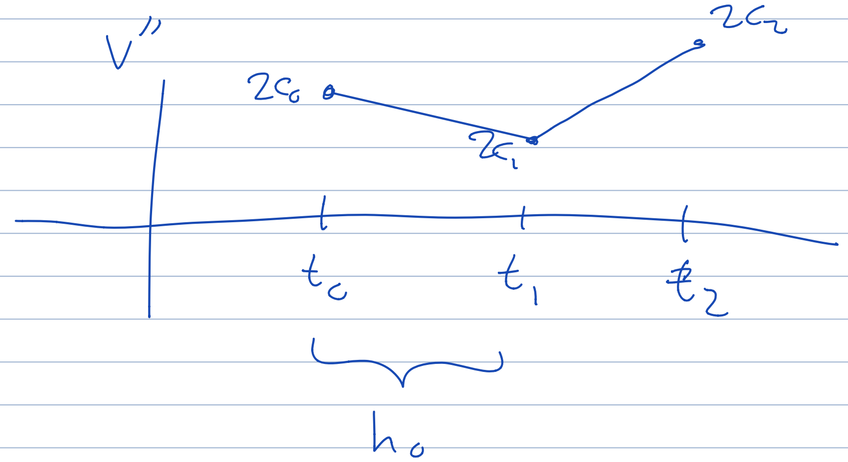
$$\parallel$$

$$2c_0 + 6d_0(t_1-t_0)$$



slope $6d_0$

$$6d_0 h_0 = 2c_1 - 2c_0$$



$$\boxed{d_0 = \frac{c_1 - c_0}{3h_0}}$$

$a_0 = S_0(t_0) = f(t_0)$ given

$\boxed{\text{$a_i$'s, d_i's known via f, $c_0, \dots, c_n$}}$