Start Ch 11, really a continuation of Ch 10...

HW 6

		28
2	4	6
9	11	

(due) not handed in or graded

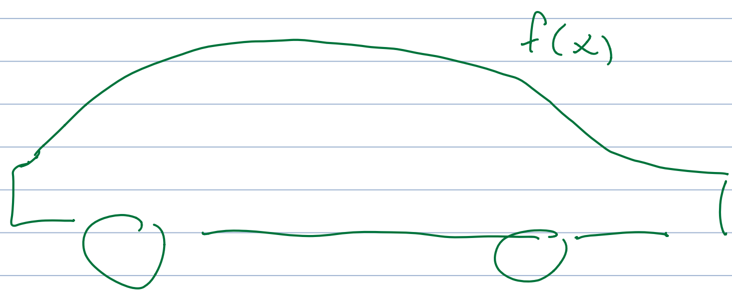
Option 1

Option 2: gradescope 11:59 Th March 5

HW 6 ~~due~~ not handed in

Ch 11: Splines, Piecewise Polynomial Interpolation

Car



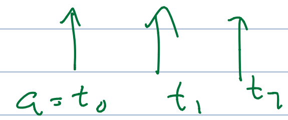
wind

But: You can evaluate $f(x)$ at any number of points you want,

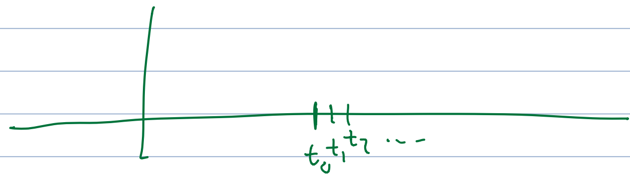
(1) You really know a lot of f: f', f'', ...

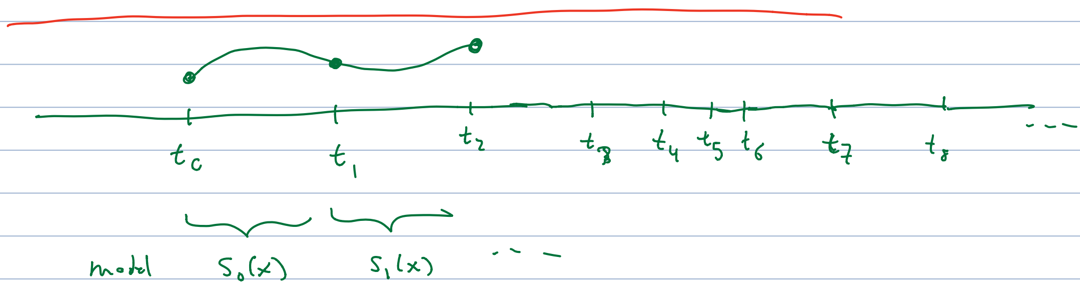
(2) " " don't know $f'(x)$ ...

Piecewise



$$a = t_0 < t_1 < \cdots < t_n = b$$





Model $f(x)$ as $\begin{cases} S_0(x) & t_0 \le x \le t_1 \\ S_1(x) & t_1 \le x \le t_2 \\ \vdots & \end{cases}$ } "piecewise polynomial interpolation"

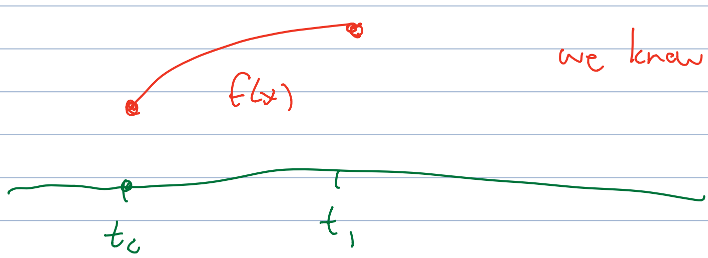
$S_0(x) = \text{poly} = c_0 + c_1 x + \cdots + c_m x^m$ $\quad$ m small

cubic $\quad c_0 + c_1 x + c_2 x^2 + c_3 x^3$

$S_1(x) = \text{poly} = \tilde{c}_0 + \hat{c}_1 x + \cdots + \hat{c}_m x^m$

cubic $\quad c_0 + c_1 x + c_2 x^2 + c_3 x^3$

Case ① we know f, f' (maybe f'')



we know $f(t_0)$ and $f'(t_0)$

$f(t_1)$ $\quad$ $f'(t_1)$

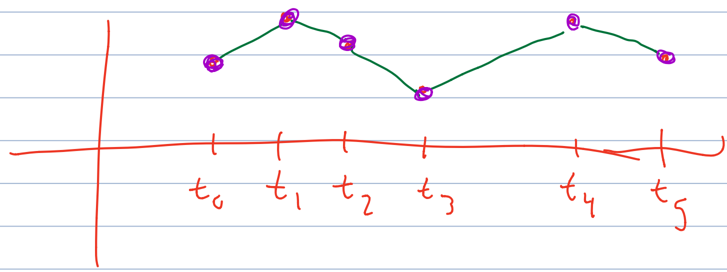
We can take: piece-by piece Hermite cubic interpolation

$S_0(x) =$ deg ≤ 3 poly s.t.

$$S_0(t_0) = f(t_0)$$
$$S_0'(t_0) = f'(t_0)$$
$$S_0(t_1) = f(t_1)$$
$$S_0'(t_1) = f'(t_1)$$

What if:

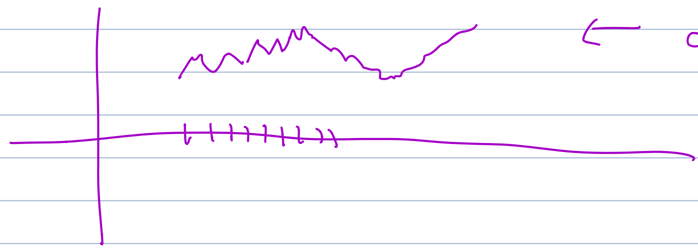
We don't know f'



"piecewise linear interp"

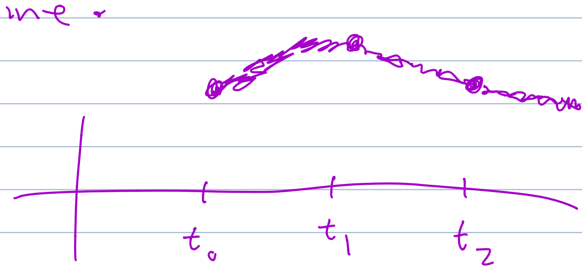
not a great model

"broken line interpolation"



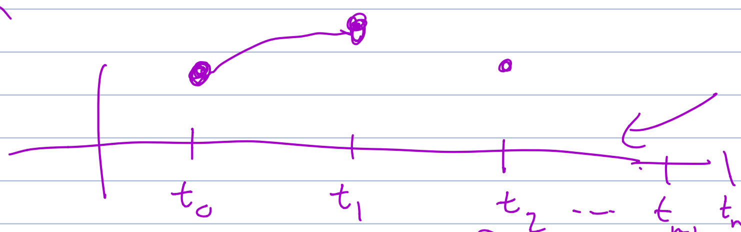
doesn't look good...

Spline:



if measured, you can't hope to know $f'(x)$...

solution



$S_2(x)$ ...

$C_{0,1} + C_{1,1}x + C_{2,1}x^2 + C_{3,1}x^3$

$=$

$S_1(x)$

$$S_0(x) = c_{0,0} + c_{1,0}x + c_{2,0}x^2 + c_{3,0}x^3$$

choose s_0

$$\left.\begin{aligned} S_0(t_0) &= f(t_0) \\ S_0(t_1) &= f(t_1) \end{aligned}\right\} \text{2 eqs}$$

$$\left.\begin{aligned} S_1(t_1) &= f(t_1) \\ S_1(t_2) &= f(t_2) \end{aligned}\right\} \text{2 eqs}$$

$\vdots$

t_0 t_1 $\rightarrow$ 4 parameters

t_1 t_2 $\rightarrow$ " "

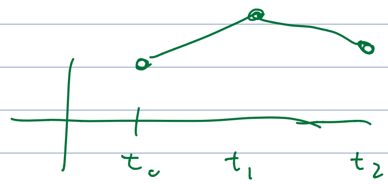
$\vdots$

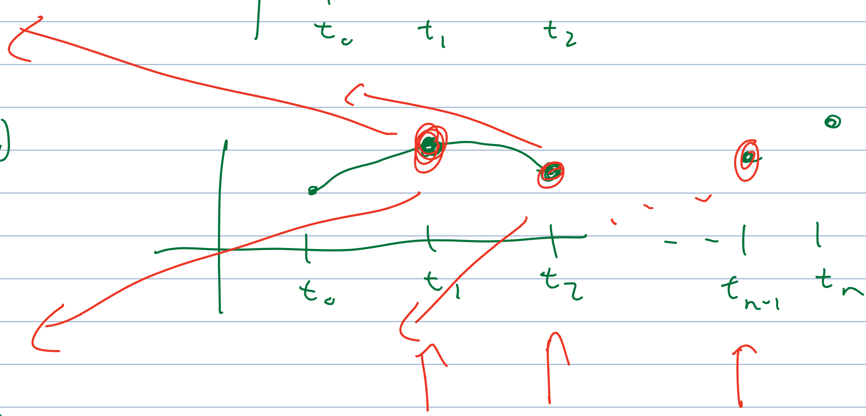
t_{n-1} t_n $\rightarrow$

4 n parameters

2n linear equations

$$S_0'(t_1) = S_1'(t_1)$$





$$S_0''(t_1) = S_1''(t_1)$$

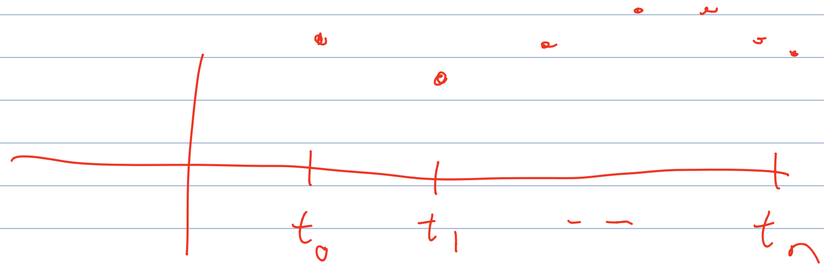
2 (n-1) equations

4n variables, $2n + 2(n-1) = 4n-2$ equations

What is special about "splines" piecewise cubic

(deg ≥ 3) that match der + 2nd derivative at

interfaces:

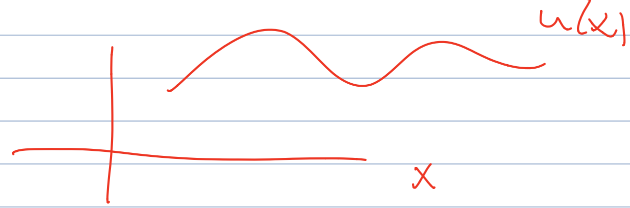


$$\left\{ \text{All } u : \mathbb{R} \to \mathbb{R} \text{ s.t. } \begin{aligned} u(t_0) &= \text{given}, f(t_0) \\ u(t_1) &= \text{given}, f(t_1) \\ &\vdots \\ u(t_n) &= \text{given}, f(t_n) \end{aligned} \right\} = \mathcal{U}$$

Claim:

$$\underbrace{\mathcal{E}_{2nd\ der}}(u) = \int_{t_0}^{t_n} \underbrace{|u''(x)|^2 \, dx}$$

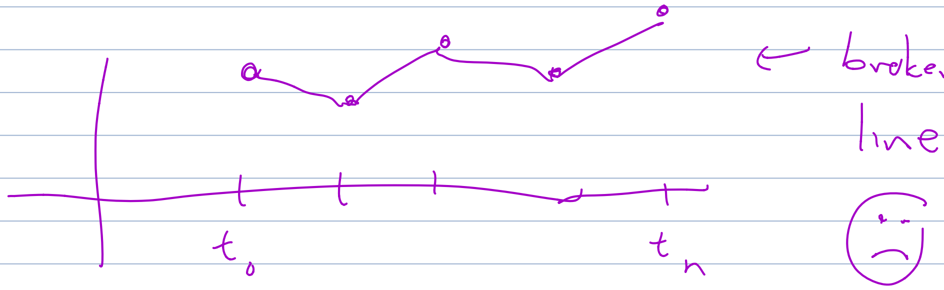
the minimizer of energy over $\mathcal{U}$ is a cubic spline



energy = ?

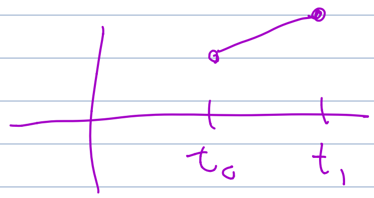
$$\mathcal{E}_{length}(u) = \int_{t_0}^{t_n} \sqrt{1 + (u')^2(x)} \, dx$$

minimize



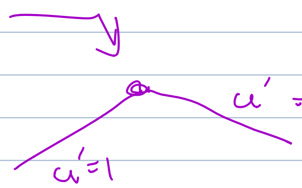
$$\mathcal{E}_{\text{Dirichlet}}(u) = \int_{t_0}^{t_n} |u'(x)|^2\, dx$$

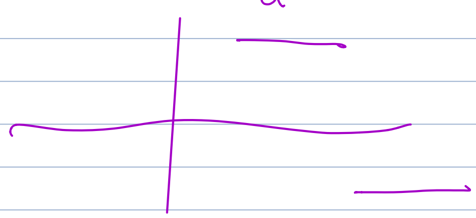
$w(\dot{x})$



Also $\displaystyle\int_{t_0}^{t_n} e^{|u'(x)|^{2020}}\, dx$

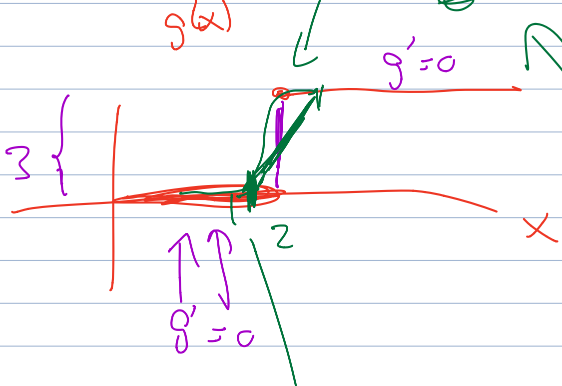
$$\mathcal{E} := \int |u''(x)|^2\, dx$$





$\int |u''|$ OK

$\int |u''|^2 = \infty$!!



$\displaystyle\int_0^\infty |g'|$ ← variation

$= 3$

$2-\varepsilon \quad 2+\varepsilon$

$$|\varphi''|^2 = \left(\frac{1}{\varepsilon}\right)^2$$

on an ~~inter~~ interval size ε