CPSC 303   Jan 20 : Starting Ch. 10 on interpolation

Today : Review slides from Friday, specifically

(1) Slide #3 : The word "why?" (why interpolation)

(2) Slide #9 : Bad Conditioning can be Bad

[Monday, Jan 13 : The "Fibonacci Lecture" ; Wednesday : snow]

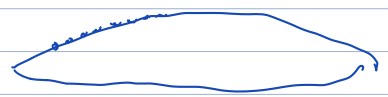
Interpolation:

- Get an exact curve fit
- Related : get an approximate for a lot of data

Setup: $n+1$ data points $(x_0, y_0), \ldots (x_n, y_n)$

model : $V(x) = c_0 \varphi_0(x) + c_1 \varphi_1(x) + \cdots + c_n \varphi_n(x)$

Chapter 10 $\rightarrow$ $c_0 + c_1 x + c_2 x^2 + \ldots + c_n x^n$.

Applications :
- Splines : Airplane, Car 
- Piecewise polynomials
- Differential Equations

Important Principles common to
- ML & big data sets
- ...

Bad (ill) conditioning can be bad...

Reasonable system:

$V(x) = c_0 + x c_1 + x^2 c_2$ parabola

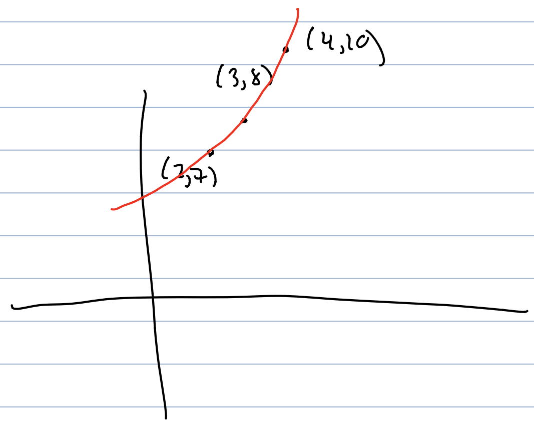
data $(2,7), (3,8), (4,10)$:

$(2,7)$: $x=2, y=7$ $c_0 + 2c_1 + 4c_2 = 7$

$(3,8)$: $c_0 + 3c_1 + 9c_2 = 8$

$(4,10)$: $c_0 + 4c_1 + 16c_2 = 10$

"reasonable"



$$c_0 + 2c_1 + 4c_2 = 7$$

$$c_0 + 3c_1 + 9c_2 = 8$$

$$(1.00001)c_0 + (3.00000004)c_1 + (8.9999992)c_2 = 10.1$$

$$(1.000000)c_0 + (3.000000)c_1 + (9.00000)c_2 = 8$$

ad hoc reason:

$$(0.00001)c_0 + (0.000\ldots04)c_1 + (-0.000\ldots08)c_2 = 2.1$$

Say: $10^{-8} c_0 + 2 \cdot 10^{-8} c_1 - 3 \cdot 10^{-12} c_2 = 3$

maybe $c_0 = 0$, $c_1 = 0$, c_2 large

If $\max(|c_0|, |c_1|, |c_2|) = M$

$$(10^{-8} + 2\cdot 10^{-8} + 3\cdot 10^{-12}) M \geq 3$$

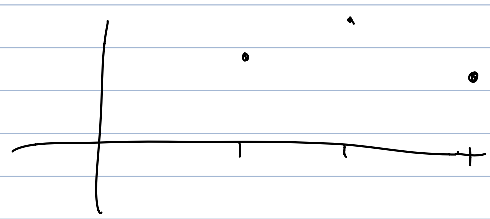
$$M \geq \frac{3}{3\cdot 10^{-8} + 10^{-12}} \text{ ... large ...}$$

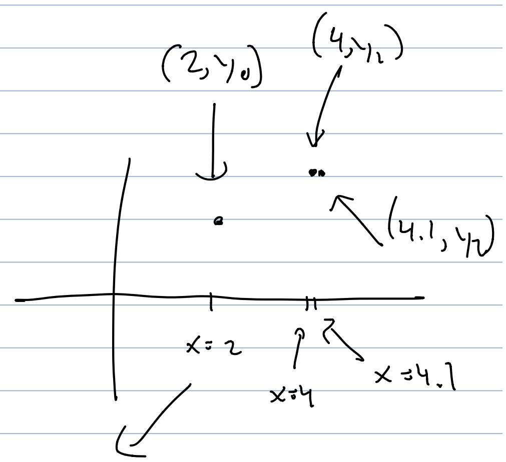
⇝ When 3rd eq is nearly the 2nd eq

OR " " " " " some combo of 1st & 2nd

⇝ "ill / bad" conditioning ⇝ (can be) bad ...

Does this arise in interpolation?





$$c_0 + 2c_1 + 4c_2 = y_0$$

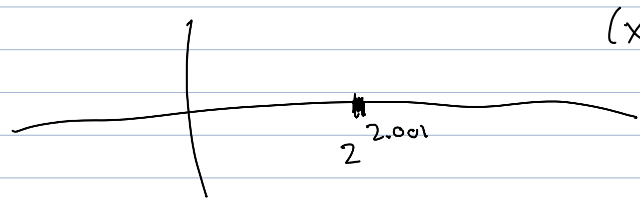
close
$$\begin{cases} c_0 + 4c_1 + 16c_2 = y_1 \\ c_0 + 4.1\,c_1 + (4.1)^2 c_2 = y_2 \end{cases}$$

$(4.1)^2 = 16.81$

Linear interpolation

$(x_0, y_0) = (2, y_0)$

$(x_1, y_1) = (2.001, y_1)$



$$c_0 + 2c_1 = y_0$$

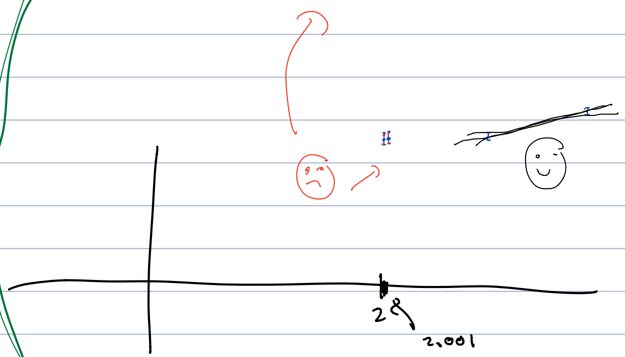
$$c_0 + (2.001)c_1 = y_1$$

y_0 = anything

4 ± 0.04

y_1 = close to 4 ± 0.04

simple type of error to consider



$$(0.001)c_1 = y_1 - y_0$$

$$c_1 = 10^3(y_1 - y_0)$$