CPSC 303 : Numerical Experiments & HW 1

& Limits of Finite Precision :

Fibonacci recurrence : $x_{n+2} = x_{n+1} + x_n$

The $(\frac{1}{2}, 1)$ recurrence : $x_{n+2} = \frac{3}{2} x_{n+1} - \frac{1}{2} x_n$

The $(\frac{1}{3}, 1)$ recurrence : $x_{n+2} = \frac{4}{3} x_{n+1} - \frac{1}{3} x_n$

Matrix form : $\begin{bmatrix} x_{n+2} \\ x_{n+1} \end{bmatrix} = \begin{bmatrix} \alpha & \beta \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_{n+1} \\ x_n \end{bmatrix}$ for $x_{n+2} = \alpha x_{n+1} + \beta x_n$

Matrix form discussed on Monday "Fibonacci Lecture"

To solve $x_{n+2} = x_{n+1} + x_n$

Guess a few nice solutions :

$$x_n = r^n \quad \text{for some } r \in \mathbb{R}, \in \mathbb{C}$$

(real) (complex)

$$r^{n+2} = r^{n+1} + r^n$$

(assume $r \neq 0$)

$$r^2 = r + 1, \quad r^2 - r - 1 = 0$$

$$r = \frac{1+\sqrt{5}}{2} \quad \text{golden ratio} \quad \sim 1.6$$

$$\frac{1-\sqrt{5}}{2} \quad \text{golden ratio \underline{conjugate}} \quad \sim -0.6$$

One solution: $\quad x_n = \left(\frac{1+\sqrt{5}}{2}\right)^n$

$$\boxed{\left(x_{n+2} = x_{n+1} + x_n \quad \forall n \in \mathbb{Z}\right) \Leftrightarrow 3x_{n+2} = 3x_{n+1} + 3x_n}$$

$x_n = \left(\frac{1+\sqrt{5}}{2}\right)^n$ is a solution, $\Rightarrow$ also $3\left(\frac{1+\sqrt{5}}{2}\right)^n$ is

In general

$$x_n = C_1 \left(\frac{1+\sqrt{5}}{2}\right)^n + C_2 \left(\frac{1-\sqrt{5}}{2}\right)^n$$

is a solution.

These are all possible solutions:

given $\boxed{x_0, x_1}$ $\quad x_2 \leftarrow x_0, x_1, \quad x_3 \leftarrow x_2, x_1, \ldots$

$x_{-1} \leftarrow x_0, x_1, \quad x_{-2} \leftarrow x_{-1}, x_0, \ldots$

also

$$x_{n+2} = x_{n+1} + x_n$$

$$y_{n+2} = y_{n+1} + y_n$$

$$(x+y)_{n+2} = (x+y)_{n+1} + (x+y)_n$$

We have

$$x_n = C_1 \left(\frac{1+\sqrt{5}}{2}\right)^n + C_2 \left(\frac{1-\sqrt{5}}{2}\right)^n$$

Fibonacci : Solve

$$\boxed{\begin{aligned} &c_1\left(\frac{1+\sqrt5}{2}\right)^0 + c_2\left(\frac{1-\sqrt5}{2}\right)^0 = x_0 = 0 \\ &c_1(\quad)^1 + c_2(\quad)^1 = x_1 = 1 \end{aligned}}$$

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get $F_n = \left(\frac{1}{\sqrt5}\right)\left(\frac{1+\sqrt5}{2}\right)^n - \left(\frac{1}{\sqrt5}\right)\left(\frac{1-\sqrt5}{2}\right)^n$

$n \to \infty$ $\quad (1.6)^n \to \infty \quad (-0.6)^n \to 0$

We have

$$x_n = c_1\left(\frac{1+\sqrt5}{2}\right)^n + c_2\left(\frac{1-\sqrt5}{2}\right)^n$$

300 $\quad$ taking to infinity $\quad$ 17.5

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Say $c_1 = 0$, $c_2 = 1$

$$x_n = \left(\frac{1-\sqrt5}{2}\right)^n = (-0.6\ldots)^n$$

$x_0 = 1$, $x_1 = \frac{1-\sqrt5}{2}$, $x_2 = \left(\frac{1-\sqrt5}{2}\right)^2, \ldots$

$c_1 = -10^{-13}$

$c_2 = 1$

$$\boxed{(-10^{-13})\left(\frac{1+\sqrt5}{2}\right)^n + 1\cdot\left(\frac{1-\sqrt5}{2}\right)^n}$$

$$x_{n+2} = \frac{3}{2} x_{n+1} - \frac{1}{2} x_n$$

$$x_n = C_1 1^n + C_2 \left(\frac{1}{2}\right)^n$$

$$\left[r^2 = \frac{3}{2} r - \frac{1}{2} \quad , \quad r = 1, \frac{1}{2} \right]$$

$$C_1 = 0$$

$$C_2 = 1$$

$$x_n = \left(\frac{1}{2}\right)^n$$