CPSC 303, Jan 8.

- For 3 classes: Recursive relations (equations) and finite pression!

Two-term RR: $x_{n+1} = 3x_n \quad \forall n \in \mathbb{Z}$

Three-term RRs: $x_{n+2} = 5x_{n+1} - 6x_n$

+ Fibonacci, some others

And relationship to
- finite precision
- ODE's
- interpolation
- etc.
- shift op

[Mention von Neumann quote]

(groups 1-3)

Admin stuff:

- group homework (vote) Passed

- gradescope with a1b2c accounts

- first HW due Jan 23, 11:59 pm (will appear by Friday)

2-term recurrence equations

$x_{n+1} = 2020\, x_n$ $\qquad n \in \mathbb{Z} = \{\dots,-2,-1,0,1,2,\dots\}$

$\mathbb{N} = \{1,2,3,\dots\}$

$\mathbb{Z}_{\geq 0} = \{0,1,2,\dots\}$

$\mathbb{R}$ = reals

$\mathbb{C}$ = complex numbers

"$x_n$"

$\dots, x_{-2}, x_{-1}, x_0, x_1, x_2, x_3, \dots$ $\qquad \{x_n\}$

$\dots, F_{-2}=-1, F_{-1}=1, F_0=0, F_1=1, F_2=1, F_3=2, \dots$

$\{x_n\}$ $\qquad \{F_n\}$

$\{x_n\}_{n\in\mathbb{Z}}$

$\mathbf{x}$, $x$ $\qquad \sigma$ = shift operator

$$\mathbf{x} \longrightarrow \sigma \mathbf{x}$$

$$\boxed{\sigma x_n = (\sigma \mathbf{x})_n = \mathbf{x}_{n+1}}$$

$$x_{n+1} = 2020\, x_n \qquad \forall n \in \mathbb{Z}$$

$x_{n}$ time

$x_0$ = initial situation of $x$

Given

$$\left.\begin{aligned} x_{n+1} &= 2020\, x_n \\ x_{n+1} - 2020\, x_n &= 0 \\ (\sigma - 2020)\, x_n &= 0 \\ (\sigma - 2020)\, \cancel{x} &= 0 \end{aligned}\right\} \text{ and } x_0 = 303 \text{ initial condition}$$

$x_0 = 303$, $\quad x_1 = 2020\, x_0 = 2020 \cdot 303$

$x_2 = 2020\, x_1 = 2020^2 \cdot 303$

$\vdots$

$$x_n = 2020^n\, x_0 \qquad (n = 0, 1, 2, \ldots, -1, -2, \ldots)$$

$$x_{n+1} = 2020\, x_n \iff x_n = \frac{x_{n+1}}{2020}$$

$$X = \begin{array}{cccc} x_{-1} & x_0 & x_1 & x_2 \\ 21 & 3 & 5 & -17 \end{array}$$

$$\sigma X = \cdots \quad x_{-1} \quad x_0 \quad x_1 \quad - -$$

$=$

Differential Eq.

$$\dot{x} = 17 x$$

$$\frac{dx}{dt}$$

$$\rightsquigarrow \quad x(t) = e^{17t} x(0)$$

$$\rightsquigarrow \quad \frac{x(t+h) - x(t)}{h} = 17 x(t)$$

"h infinitesimal"

$$x(t+h) \approx (1 + 17h) x(t)$$

$$x(t+2h) = (1+17h)^2 x(t)$$

$$x(t+3h) = (1+17h)^3 x(t)$$

$$x\left(t+\left(\frac{1}{h}\right)h\right) = (1+17h)^{1/h}\, x(t)$$

$$\uparrow \qquad\qquad\qquad \uparrow$$

$$x(t+1) \qquad h \to 0 \qquad e^{17}\, x(t)$$

$$\left( x_n = 2020^n x_0 \right)$$

$n \in \mathbb{Z}$

=

Fibonacci recurrence

$$x_{n+2} = x_{n+1} + x_n$$

Before this:

$$x_{n+2} = 5x_{n+1} - 6x_n$$

(has 2 parameters, 2 dim vect sp)

given "initial" conditions $x_0 = 1, x_1 = 1$

$x_0 = 1, \; x_1 = 1, \; x_2 = 5x_1 - 6x_0 = -1$

$x_3 = 5x_2 - 6x_1 = 5(-1) - 6(1) = -11$

$x_4 = 5x_3 - 6x_2 = 5(-11) - 6(-1) = -49$

$x_{-1}$: $\quad x_1 = 5x_0 - 6x_{-1}$

$1 = 5 - 6x_{-1}, \quad x_{-1} = \frac{4}{6}, \ldots$

$x_n = ???$

Simple solutions to

$$x_{n+2} - 5x_{n+1} + 6x_n = 0$$

(1) $x_n = 0$

$$\begin{array}{ccccc} & x_{-1} & x_0 & x_1 & x_2 \\ & & \| & \| & \| \\ \ldots, & r^{-1}, & 1, & r, & r^2, \ldots \end{array}$$

$x_n = r^n$ for some $r \in \mathbb{R}, \mathbb{C}$ $\quad (r \neq 0)$

$$r^{n+2} - 5r^{n+1} + 6r^n = 0$$

$$r^2 - 5r + 6 = 0$$

$$r = 2, 3$$

(2) $\ldots, \frac{1}{4}, \frac{1}{2}, 1, 2, 2^2, \ldots$ $x_n = 2^n$

(3) $\ldots, \frac{1}{9}, \frac{1}{3}, 1, 3, 3^2, \ldots$ $x_n = 3^n$

(4) also $\boxed{x_n = c_1 2^n + c_2 3^n}$

$$\boxed{x_{n+2} - 5x_{n+1} + 6x_n = 0}$$

fit to initial cond

$$\boxed{\begin{aligned} (1 &= x_0 = c_1 + c_2) \cdot 2 \\ 1 &= x_1 = c_1 \cdot 2 + c_2 \cdot 3 \end{aligned}}$$

$$c_2 = -1, \quad c_1 = 2$$

$$2 \cdot 2^n + (-1) 3^n$$