CPSC 303: Numerical Methods ... ODE (calculus), also splines

↪ classical

↪ machine learning gradient searches

- Admin:

Grading scheme:

$55\% \cdot f + 35\% \max(m, f) + 10\% \max(h, m, f)$

h = homework, m = midterm, f = final

- Homework: Rough 8 homework assignment.

Due probably: Thursdays at 11:59 pm (local time)

Likely some MATLAB computations, some theory...

First assignment probably due Jan 23 (or so).

Website: www.cs.ubc.ca/~jf

Use piazza, probably use gradescope, (free)

Textbook: Ascher & Greif

1st two weeks:

Finite Precision — Supply article ← related to Ch 2

Curve Fitting — Supply article ← " " Ch 10

~~In~~Interpolation = Exact Curve Fitting — " ← motivation " "

Topic: Finite precision (CL 2 [A & G]):

Upshot:

(1) MATLAB work in double precision

(2) There is an IEEE standard for what is double precision — every modern computer should be doing the same thing: round, $\sin(x)$, add, multiply, etc.

(3) Roughly — double precision 64-bit representation of a real number; base 2 scientific notation,

$\text{blah} \cdot 2^{\text{blah}'}$ (not $3{,}179 \times 10^5$)

roughly 14–16 sig digits, roughly $10^{\pm 300}$

3,179 base 10, base 1.0110011010101

(4) There are good reasons to have a uniform standard

Test things:

3 term recursion: $\ldots, F_{-3} = 2, F_{-2} = -1, F_{-1} = 1,$

$F_0 = 0, F_1 = 1, F_2 = 1, F_3 = 2, F_4 = 3, F_5 = 5, F_6 = 8,$

$F_7 = 13, F_8 = 21, \ldots$

$$F_{n+2} = F_{n+1} + F_n$$ "Fibonacci numbers"

$$F_n = \left(\frac{1}{\sqrt{5}}\right)\left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{-1}{\sqrt{5}}\right)\left(\frac{1-\sqrt{5}}{2}\right)^n$$

$$F_n = F_{n+2} - F_{n+1}$$

In general:

$$X_{n+2} - X_{n+1} - X_n = 0$$

for all $n \in \mathbb{Z} = \{\ldots -2, -1, 0, 1, 2 \ldots\}$

$$X_n = C_1\left(\frac{1+\sqrt{5}}{2}\right)^n + C_2\left(\frac{1-\sqrt{5}}{2}\right)^n$$

=

If take $C_1 = 0$, $C_2 = 1$

$$X_n = \left(\frac{1-\sqrt{5}}{2}\right)^n$$

$X_0 = 1$, $X_1 = \frac{1-\sqrt{5}}{2}$, $X_2 = \left(\frac{1-\sqrt{5}}{2}\right)^n$, ...

Related:

$$x_n = \left(\tfrac{4}{3}\right) x_{n-1} - \left(\tfrac{1}{3}\right) x_{n-2}$$

claim:

$$x_0 = 1, \quad x_1 = \frac{1}{3}, \quad x_2 = \frac{1}{9}, \quad x_3 = \frac{1}{27}, \ldots$$

i.e.

$x_n = \left(\frac{1}{3}\right)^n$ is a solution

$x_n = 1^n = 1$ is a solution

Homework: $\left(\sigma - \frac{1}{3}\right)(\sigma - 1) = \sigma^2 - \frac{4}{3}\sigma + \frac{1}{3}$

$\sigma(x_n) = x_{n+1}$
shift operator

$$x_{n+2} - \frac{4}{3} x_{n+1} + x_n = 0$$

What happens?

$x_0 = 1$, $x_1 = \frac{1}{3}$, and $x_{n+2} = \frac{4}{3} x_{n+1} - \frac{1}{3} x_n$

$n \geq 0$

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