CPSC 303: MIDTERM PRACTICE QUESTIONS, SET 3

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All questions on Homework 1–6 should be considered midterm practice.

(1) Define for each sequence

$$\mathbf{y} = \{y_n\}_{n\in\mathbb{Z}} = \{\dots, y_{-1}, y_0, y_1, y_2, \dots\}$$

a new sequence $D\mathbf{y}$ (known as the "(forward) difference of $\mathbf{y}$") defined by

$$(D\mathbf{y})_n = y_{n+1} - y_n.$$

(a) Show that if $\mathbf{y}$ is given by $y_n = n^3$, then $D\mathbf{y}$ is a polynomial in n of degree 2 and leadning term $3n^2$.
(b) Show that if $\mathbf{y}$ is given by $y_n = n^4$, then $D\mathbf{y}$ is a polynomial in n of degree 3 and leadning term $4n^3$.
(c) Show that if $\mathbf{y}$ is given by $y_n = n^4$, then $D^2\mathbf{y}$ is a polynomial in n of degree 2 and leadning term $12n^2$.

(2) Define for each sequence

$$\mathbf{y} = \{y_n\}_{n\in\mathbb{Z}} = \{\dots, y_{-1}, y_0, y_1, y_2, \dots\}$$

a new sequence $D\mathbf{y}$ (known as the "(forward) difference of $\mathbf{y}$") defined by

$$(D\mathbf{y})_n = y_{n+1} - y_n.$$

(a) Show that $D^3\mathbf{y}$ is the sequence whose n-th term is

$$y_{n+3} - 3y_{n+2} + 3y_{n+1} - y_n.$$

(b) Show that the sequences $\mathbf{y}$ given by either $y_n = 1$ or $y_n = n$ or $y_n = n^2$ all satisfy $D^3\mathbf{y} = 0$.
(c) Show that for any $C_1, C_2, C_3 \in \mathbb{R}$, the sequence $\mathbf{y}$ given by either $y_n = C_1 + C_2 n + C_3 n^2$ all satisfy $D^3\mathbf{y} = 0$.
(d) Show that the sequence $\mathbf{y}$ given by $y_n = n^3$ satisfies $D^4\mathbf{y} = 0$.
(e) Show that for any C_1, C_2, C_3, C_4 the sequence given as $y_n = C_1 + C_2 n + C_3 n^2 + C_4 n^3$ satisfies $D^4\mathbf{y} = 0$.

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(f) Explain why for any y_0, y_1, y_2, y_3 there exist $C_1, \ldots, C_4$ that satisfy

$$y_n = C_1 + C_2 n + C_3 n^2 + C_4 n^3$$

for $n = 0, 1, 2, 3$.

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