CPSC 303: MIDTERM PRACTICE QUESTIONS, SET 1

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All questions on Homework 1–6 should be considered midterm practice.

(1) What is the largest normal positive number in double precision?

(2) What is the smallest normal positive number in double precision?

(3) What is the largest subnormal positive number in double precision?

(4) What is the smallest subnormal positive number in double precision?

(5) In double precision, are the values of:

$$\texttt{Inf} - \texttt{Inf}, \quad \texttt{Inf} + \texttt{Inf}, \quad \texttt{Inf} + 2020.$$

(6) **Circle the correct numeral (i, ii, iii, or iv) in each of the following questions.**
  (a) Normal numbers in double precision have roughly:
    (i) 530 bits of precision
    (ii) 53 bits of precision
    (iii) 530 digits of precision
    (iv) 53 digits of precision
  (b) In double precision, a number that is 10^7 (which is roughly 2^{23}) times larger than 2^{-1074} is stored with roughly
    (i) 7 bits of precision
    (ii) 7 digits of precision
    (iii) 2^7 bits of precision
    (iv) 53 bits of precision
  (c) In double precision, a number that is roughly 2^9 times larger than 2^{-1074} is stored with roughly
    (i) 9 bits of precision
    (ii) 9 digits of precision
    (iii) 2^9 bits of precision
    (iv) 53 bits of precision

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(d) In double precision, a number that is roughly 2^{100} times larger than 2^{-1074} is stored with roughly
  (i) 100 bits of precision
  (ii) 100 digits of precision
  (iii) 2^{100} bits of precision
  (iv) 53 bits of precision

(e) In double precision, a number that is roughly 2^{100000} times larger than 2^{-1074} is stored with roughly
  (i) 100 bits of precision
  (ii) 100 digits of precision
  (iii) 2^{100} bits of precision
  (iv) none of the above

(f) In double precision, the expression 3^m number is evaluated exactly for a postive integer m iff
  (i) $3^m \le 2^{53} - 1$
  (ii) $m \le 53$
  (iii) $3^m \le 2^{1024} - 1$.
  (iv) $3^m \le 2^{1023}(2 - 2^{-52})$.

(7) Consider the recurrence

$$x_{n+2} = 2x_{n+1} + x_n.$$

(a) What is the general solution of this equation? Write this in the form $C_1 r_1^n + C_2 r_2^n$ for some $r_1, r_2 \in \mathbb{R}$.

(b) What is the solution to this recurrence in the case $x_0 = 1$ and $x_1 = 1 - \sqrt{2}$?

(c) If you numerically compute the values of $x_2, x_3, \ldots$ of this recurrence starting with $x_0 = 1$ and $x_1 = 1 - \sqrt{2}$, for n large what will $|x_n|$ be reported as:
  (i) Inf
  (ii) 0 or some periodically repeating sequence of subnormal numbers.
  (iii) Nan
  (iv) 10^m where m is roughly between -15 and -19.

(d) What is the solution to this recurrence in the case $x_0 = 1$ and $x_1 = 1 + \sqrt{2}$?

(e) If you numerically compute the values of $x_2, x_3, \ldots$ of this recurrence starting with $x_0 = 1$ and $x_1 = 1 + \sqrt{2}$, for n large what will $|x_n|$ be reported as:
  (i) Inf
  (ii) 0 or some periodically repeating sequence of subnormal numbers.
  (iii) Nan
  (iv) 10^m where m is roughly between -15 and -19.

(8) Consider the recurrence

$$x_{n+2} = (8/7)x_{n+1} - (1/7)x_n.$$

(a) What is the general solution of this equation? Write this in the form $C_1 r_1^n + C_2 r_2^n$ for some $r_1, r_2 \in \mathbb{R}$.

(b) What is the solution to this recurrence in the case $x_0 = 1$ and $x_1 = 1/7$?

(c) This part is **multiple choice**: If you numerically compute the values of $x_2, x_3, \ldots$ of this recurrence starting with $x_0 = 1$ and $x_1 = 1/7$, for n large what will $|x_n|$ be reported as:
   (i) Inf
   (ii) 0 or some periodically repeating sequence of subnormal numbers.
   (iii) Nan
   (iv) 10^m where m is roughly between -15 and -19.

(9) Consider the recurrence

$$x_{n+2} = (9/8)x_{n+1} - (1/8)x_n.$$

(a) What is the general solution of this equation? Write this in the form $C_1 r_1^n + C_2 r_2^n$ for some $r_1, r_2 \in \mathbb{R}$.

(b) What is the solution to this recurrence in the case $x_0 = 1$ and $x_1 = 1/8$?

(c) This part is **multiple choice**: If you numerically compute the values of $x_2, x_3, \ldots$ of this recurrence starting with $x_0 = 1$ and $x_1 = 1/8$ [corrected on March 3, older version showed $x_1 = 1 - \sqrt{2}$], for n large what will $|x_n|$ be reported as:
   (i) Inf
   (ii) 0 or some periodically repeating sequence of subnormal numbers.
   (iii) Nan
   (iv) 10^m where m is roughly between -15 and -19.

(10) Consider the recurrence

$$x_{n+2} = x_{n+1} + 6x_n.$$

(a) What is the general solution of this equation? Write this in the form $C_1 r_1^n + C_2 r_2^n$ for some $r_1, r_2 \in \mathbb{R}$.

(b) What is the solution to this recurrence in the case $x_0 = 2$ and $x_1 = 1$?

(c) This part is **multiple choice**: If you numerically compute the values of $x_2, x_3, \ldots$ of this recurrence starting with $x_0 = 2$ and $x_1 = 1$, for n large what will $|x_n|$ be reported as:
   (i) Inf
   (ii) 0 or some periodically repeating sequence of subnormal numbers.
   (iii) Nan
   (iv) 10^m where m is roughly between -15 and -19.

(11) Find an exact formula for

$$\begin{bmatrix} 1 & 6 \\ 1 & 0 \end{bmatrix}^n \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

and justify your formula. [Hint: rather than diagonalize the above 2×2 matrix, it may be easier to relate this to a recurrence equation.]

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