HOMEWORK 2, CPSC 303, SPRING 2020

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Please note:

(1) You must justify all answers; no credit is given for a correct answer without justification.
(2) Proofs should be written out formally.
(3) Homework that is difficult to read may not be graded.
(4) You may submit homework in groups of 1-3. You must acknowledge any sources (e.g., Wikipedia, other books, other articles) you have used beyond the textbook and the article(s) on the class website.

In these exercises, "the handout" refers to the article "Recurrence Relations and Finite Precision" on the CPSC 303 homepage.

Write you solutions to the following problems concisely. Marks will be deducted if you submit 80 values—or even 40 values—of any output the MATLAB code below produces.

(1) Let

$$A(n) = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n$$

(a) Consider the following MATLAB code for computing some values of $A(n)$:

```
A = [ 1 , 1 ; 1 , 0 ]
for n=1:8, n, A^n , end
```

Write down an exact formula for A^n based on what you see.

(b) What does the following MATLAB code do?

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```
A = [ 1 , 1 ; 1 , 0 ]
for n=1:8, n, A=A^2 , end
```

(c) Show that the equation $x_{n+2} = x_{n+1} + x_n$ is equivalent to the equation

$$\begin{bmatrix} x_{n+2} \\ x_{n+1} \end{bmatrix} = A \begin{bmatrix} x_{n+1} \\ x_n \end{bmatrix}$$

(d) Describe the exact value of

$$A^n \mathbf{v}, \quad \text{where} \quad v = \begin{bmatrix} \left(1 - \sqrt{5}\right)/2 \\ 1 \end{bmatrix}$$

(e) Describe the MATLAB computation of the sequence $A^n v$ with the following code:

```
A = [ 1 , 1 ; 1 , 0 ]
v = [ (1-sqrt(5))/2 ; 1 ]
for n=1:80, n, v = A*v  , end
```

How does the output behave for various values of n?

(f) Describe the output of the following MATLAB computation, and explain (roughly) why you see these results.

```
A = [ 1 , 1 ; 1 , 0 ]
v = [ (1-sqrt(5))/2 ; 1 ]
for n=1:80, v = A*v  ; v_ratio(n) = v(1)/v(2); end
v_ratio
```

(2) Set $x_0 = 1$, $x_1 = 1/8$, and set

$$\mathbf{v}_n = \begin{bmatrix} 9/8 & -1/8 \\ 1 & 0 \end{bmatrix}^n \begin{bmatrix} x_1 \\ x_0 \end{bmatrix}$$

(a) What is the value of $\mathbf{v}_n$ in exact arithmetic?

(b) What happens with the MATLAB floating point computation of $\mathbf{v}_n$ below?

```
A = [ 9/8 , -1/8 ; 1 , 0 ]
v = [ 1/8 ; 1 ]
for n=1:400, n, v = A*v ,  end
```

(3) Set $x_0 = 1$, $x_1 = 1/7$, and set

$$\mathbf{v}_n = \begin{bmatrix} 8/7 & -1/7 \\ 1 & 0 \end{bmatrix}^n \begin{bmatrix} x_1 \\ x_0 \end{bmatrix}$$

(a) What is the value of $\mathbf{v}_n$ in exact arithmetic?

(b) Explain what you see in the MATLAB floating point computation of $\mathbf{v}_n$ below, and (roughly) why you see it.

```
A = [ 8/7 , -1/7 ; 1 , 0 ]
v = [ 1/7 ; 1 ]
for n=1:40, n, v = A*v ,  end
```

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