

HOMework 1, CPSC 303, SPRING 2020

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Please note:

- (1) You must justify all answers; no credit is given for a correct answer without justification.
- (2) Proofs should be written out formally.
- (3) Homework that is difficult to read may not be graded.
- (4) You may submit homework in groups of 1-3. You must acknowledge any sources (e.g., Wikipedia, other books, other articles) you have used beyond the textbook and the article(s) on the class website.

In these exercises, “the handout” refers to the article “Recurrence Relations and Finite Precision” on the CPSC 303 homepage.

In these exercises you should give your MATLAB code (it should be brief, like the examples in Section 5 of the handout), and you should describe the numerical sequences you produce, by **explaining** in words what is happening, and **giving a few successive values** of the sequence when something interesting happens **that you cannot adequately describe in words**.

Do not submit the entire sequence your code produces; you list between 0 to 15 values of your sequence per homework problem. Marks will be deducted if you submit 1200 values—or even 50 values—of the sequence.

- (1) Consider the following experiment: set $x_0 = 1$, $x_1 = 1/2$, and define x_2, x_3, \dots by the formula

$$x_{n+2} = (3/2)x_{n+1} - (1/2)x_n.$$

Hence in exact arithmetic, $x_n = (1/2)^n$. Perform this experiment in MATLAB, to numerically compute $x_2, x_3, \dots, x_{1200}$. At one point does this sequence cycle (repeat a pattern)? What is the pattern and what is its period?

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[Hint: Look at Section 5.4 of the handout; you could cut and paste the code from there (you may need to do this line by line). If your numerical experiment is like mine, for x_n with n roughly 1075 you will see a repeating pattern

```
{[4.9407e-324]}    {[9.8813e-324]}    {[1.4822e-323]}    {[1.4822e-323]}
{[9.8813e-324]}    {[4.9407e-324]}    {[4.9407e-324]}    {[9.8813e-324]}
{[1.4822e-323]}    {[1.4822e-323]}    {[9.8813e-324]}    {[4.9407e-324]}
```

and so the period of this repeating pattern is 6.]

- (2) Same question as (1), except with $x_0 = 1$, $x_1 = 1/3$, and

$$x_{n+2} = (4/3)x_{n+1} - (1/3)x_n$$

for $n = 0, 1, 2, \dots$

- (3) Same questions as (1) and (2), except with $x_0 = 1$, $x_1 = r$, and

$$x_{n+2} = (1+r)x_{n+1} - (r)x_n$$

for $n = 0, 1, 2, \dots$, for the value $r = 1/4$.

- (4) Same question as (3) for $r = 1/5$.

- (5) (a) In MATLAB, perform the numerical experiment $x_0 = 1$, $x_1 = 1/3$, and

$$x_{n+2} = (10/3)x_{n+1} - x_n$$

(for $n = 0, 1, 2, \dots$). Describe the results (e.g., what happens for some iterations the beginning, at the end, and somewhere where a transition occurs).

- (b) What is the general solution to the recurrence $x_{n+2} = (10/3)x_{n+1} - x_n$?
 (c) Explain why this general solution produces a numerical pattern like the one you see.

- (6) Same question as the previous question, for $x_0 = 1$, $x_1 = -1/3$, and the recurrence

$$x_{n+2} = (8/3)x_{n+1} + x_n.$$

In addition:

- (a) Type the following calculations into MATLAB

```
(-4)^100001
```

```
(-4)^100000
```

```
(-4)^100001 + (-4)^100000
```

Explain (in 3-10 words) what the terms -Inf, Inf mean, and explain (in 5-15 words) why when you add -Inf and Inf you should get NaN (not a number).

- (b) Why does the sequence in Problem (5) end in NaN repeating, while the one in Problem (6) ends in Inf (or possibly -Inf) repeating?

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