Fractional Factorial Experiments

Confounding (pg 230)

Recall, **experimental effects** are a change in mean response between factor level combinations.

Confounded effects: Two (or more) effects are confounded (aliased) if their calculated values can only be attributed to their *combined* influence rather than their unique individual influence.

Confounding

Factor–Level Combination	Effects Representation* A	B	C	Shift	Observed Response
1	−1	−1	−1	1	$y_{111} + s_1$
2	−1	−1	1	1	$y_{112} + s_1$
3	−1	1	−1	1	$y_{121} + s_1$
4	−1	1	1	1	$y_{122} + s_1$
				Average	$y_{1..} + s_1$
5	1	−1	−1	2	$y_{211} + s_2$
6	1	−1	1	2	$y_{212} + s_2$
7	1	1	−1	2	$y_{221} + s_2$
8	1	1	1	2	$y_{222} + s_2$
				Average	$y_{2..} + s_2$

*A = Temperature (−1 = 160°C, 1 = 180°C); B = Concentration (−1 = 20%, 1 = 40%); C = Catalyst (−1 = C_1, 1 = C_2).

Confounding

$$\begin{aligned} M(A) &= (\bar{y}_{2\bullet\bullet} + s_2) - (\bar{y}_{1\bullet\bullet} + s_1) \\ &= \bar{y}_{2\bullet\bullet} - \bar{y}_{1\bullet\bullet} + s_2 - s_1 \end{aligned}$$

So the calculated effect for temperature is the main effect plus the shift effect. The effects are confounded. Whereas effect for concentration:

$$\begin{aligned} M(B) &= \frac{1}{4}[(y_{121} + s_1) + (y_{122} + s_1) + (y_{221} + s_2) \\ &\quad + (y_{222} + s_2) - (y_{111} + s_1) - (y_{112} + s_1) \\ &\quad - (y_{211} + s_2) - (y_{212} + s_2)] \\ &= \bar{y}_{\bullet 2\bullet} - \bar{y}_{\bullet 1\bullet} \end{aligned}$$

is unconfounded.

Design Resolution (pg. 237)

An experimental design is of **resolution** R if all effects containing s or fewer factors are unconfounded with any effects containing fewer than $R-s$ factors.

So a design in which the main effects are not confounded with each other, but are confounded with two-factor and higher interactions is resolution-III (R_{III}).

This concept is intended to (roughly) communicate whether an experimental design allows for the estimation of all important effects without confounding.

Skipping (pg. 239—255)

Next the book spends a good deal of time talking about two-level factors in fractional experiments of powers of 2 and three-level factors in fractional experiments of powers of 3. Lets skip this.

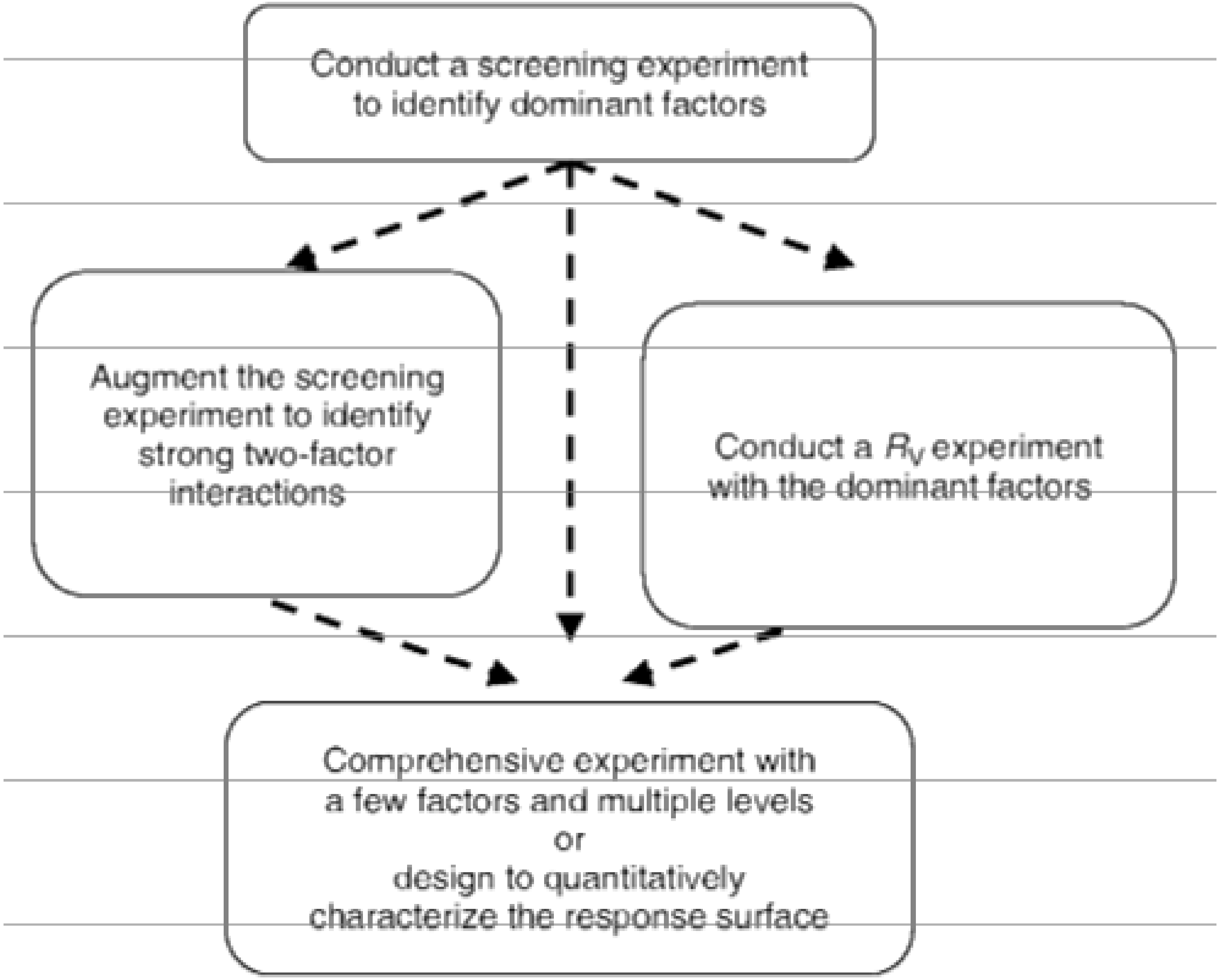
Screening Experiments (pg. 256)

Screening experiments:

- Used when a large number of factors are to be investigated but resources limit number of tests.
- Identify a small number of dominant factors, often with intent to perform future investigation.

This sounds appropriate for computer experiments, however the book says nothing concrete about how to do screening experiments in general. However, a straightforward way to do this is by just running experiments with low resolution and many factor levels.

Screening Experiments (pg. 259)



Analysis (pg 271)

We must modify the ANOVA procedure of chapter 6 to account for:

- factor level combinations which never occur,
- and unequal numbers of repeat tests.

Does MATLAB's ANOVA handle fractional experiments?

Subset Hierarchical Models (pg. 272)

Recall, **Hierarchical Models** contains only hierarchical terms (an interaction is present only if its individual factors are).

Two hierarchical models are **Subset Hierarchical Models** (SHM) if the terms in one are subsets of terms in the other.

Two SHM's can be compared with respect to their ability to adequately account for variation in response.

Error Sum of Squares (pg. 273)

Assume M_1 and M_2 are SHM's with terms in M_2 a subset of terms in M_1. The error sums of squares are SSE_1 and SSE_2 respectively. Then the reduction in error sums of squares is defined as:

$$R(M_1|M_2) = SSE_2 - SSE_1$$

This can be used to assess the effects of model factors and covariates. In complete factorial experiments the RESS is exactly equal to the sums of squares.

Error Sum of Squares (pg. 274)

Temperature (°C)	Concentration (%)	Catalyst	Yield (g)
160	20	C_1	59
		C_2	50,54
	40	C_1	
		C_2	46,44
180	20	C_1	74,70
		C_2	81
	40	C_1	69
		C_2	79

Error Sum of Squares (pg. 274)

Reduction in Error Sums of Squares

Source of Variation	df	Sum of Squares	Mean Square	F-Value
(a) With Temperature × Catalyst				
Model	6	1682.40	280.40	46.73
Error	3	18.00	6.00	
Total	9	1700.40		
(b) Without Temperature × Catalyst				
Model	5	1597.07	319.41	12.36
Error	4	103.33	25.83	
Total	9	1700.40		
(c) Temperature × Catalyst F-Statistic				
Temperature × catalyst	1	85.33	85.33	14.22
Error	3	18.00	6.00	

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“Rather than discuss formulas that would be appropriate for only this example, we rely on computer software to fit models to these data.”