STOCHASTIC LOCAL SEARCH
FOUNDATIONS AND APPLICATIONS

# Generalised Local Search Machines

Holger H. Hoos & Thomas Stützle

# Outline

1. The Basic GLSM Model
2. State, Transition and Machine Types
3. Modelling SLS Methods Using GLSMs
4. Extensions of the Basic GLSM Model

# The Basic GLSM Model

Many high-performance SLS methods are based on combinations of *simple (pure) search strategies* (*e.g.*, ILS, MA).

These hybrid SLS methods operate on two levels:

- **lower level:** execution of underlying simple search strategies
- **higher level:** activation of and transition between lower-level search strategies.

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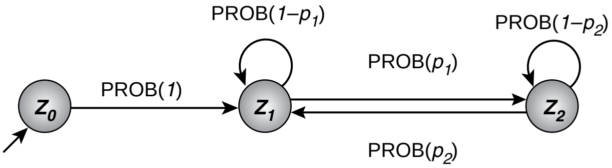
## Example: Simple 3-state GLSM




- States $z_0, z_1, z_2$ represent simple search strategies, such as Random Picking (for initialisation), Iterative Best Improvement and Uninformed Random Walk.
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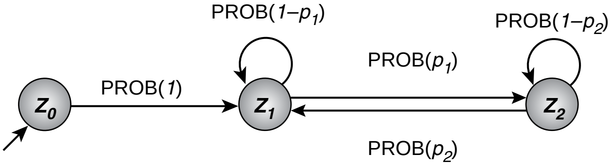


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## Generalised Local Search Machines (GLSMs)

- States $\cong$ simple search strategies.
- State transitions $\cong$ search control.

- GLSM $\mathcal{M}$ starts in initial state.
- In each iteration:
  - $\mathcal{M}$ executes one search step associated with its current state $z$;
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## Formal definition of a GLSM

A *Generalised Local Search Machine* is defined as a tuple $\mathcal{M} := (Z, z_0, M, m_0, \Delta, \sigma_Z, \sigma_\Delta, \tau_Z, \tau_\Delta)$ where:

- $Z$ is a set of states;
- $z_0 \in Z$ is the *initial state*;
- $M$ is a set of *memory states* (as in SLS definition);
- $m_0$ is the *initial memory state* (as in SLS definition);
- $\Delta \subseteq Z \times Z$ is the *transition relation*;
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- $\tau_Z : Z \mapsto \sigma_Z$ and $\tau_\Delta : \Delta \mapsto \sigma_\Delta$ associate every state $z$ and transition $(z, z')$ with a state type $\sigma_Z(z)$ and transition type $\tau_\Delta((z, z'))$, respectively.

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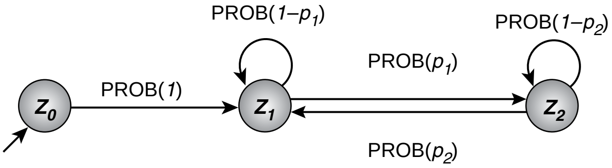


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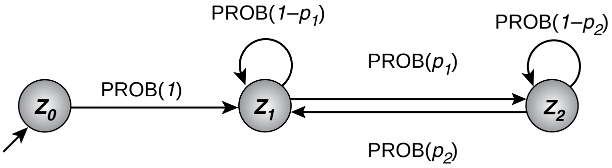


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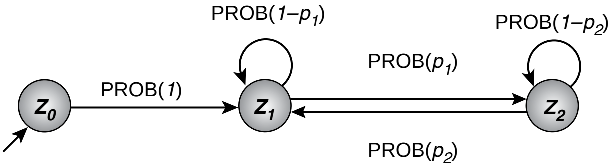


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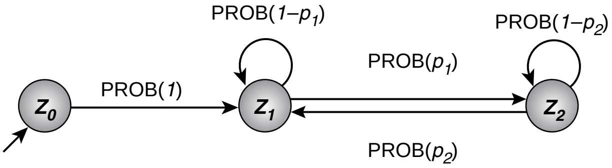


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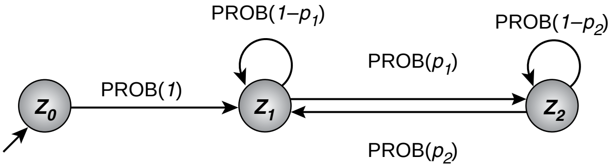


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- Start in initial state $z_0$, memory state $m_0$ (never changes).
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- *States types* formally represent (subsidiary) search strategies, whose definition is not part of the GLSM definition.
- *Transition types* formally represent mechanisms used for switching between GLSM states.
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Behaviour of a GLSM is specified by *machine definition* $+$ *run-time environment* comprising specifications of

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- Given GLSM represents the way in which *initialisation* and *step function* of a hybrid SLS method are composed from respective functions of *subsidiary component SLS methods*.

- When modelling hybrid SLS methods using GLSMs, *subsidiary SLS methods* should be as simple and pure as possible, leaving *search control* to be represented explicitly at the GLSM level.

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# State, Transition and Machine Types

In order to completely specify the search method represented by a given GLSM, we need to define:

- the GLSM model (states, transitions, ...);
- the search method associated with each *state type*, *i.e.*, step functions for the respective subsidiary SLS methods;
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## State types

- State type semantics are often most conveniently specified procedurally (see algorithm outlines for 'simple SLS methods' from Chapter 2).

- *initialising state type* = state type $\tau$ for which search position after one $\tau$ step is independent of search position before step.

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- *Unconditional deterministic transitions* – type *DET*:
  - executed always and independently of memory state or search position;
  - every GLSM state can have at most one outgoing DET transition;
  - frequently used for leaving initialising states.
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Note:

- DET transitions are a special case of PROB transitions.
- For a GLSM $\mathcal{M}$ any state that can be reached from initial state $z_0$ by following a chain of PROB($p$) transitions with $p > 0$ will eventually be reached with arbitrarily high probability in any sufficiently long run of $\mathcal{M}$.
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## Transitions types (2)

- *Conditional probabilistic transitions* – type *CPROB($C$, $p$)*:
  - executed with probability proportional to $p$ iff *condition predicate $C$* is satisfied;
  - all CPROB transitions from the current GLSM state whose condition predicates are not satisfied are *blocked*, *i.e.*, cannot be executed.

### Note:

- Special cases of CPROB($C, p$) transitions:
  - PROB($p$) transitions;
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- Condition predicates should be efficiently computable (ideally: $\leq$ linear time w.r.t. size of given problem instance).

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## Commonly used simple condition predicates:

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| $\top$ | always true |
| count($k$) | total number of GLSM steps $\geq k$ |
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All based on local information; can also be used in negated form.

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- *1-state machines*:
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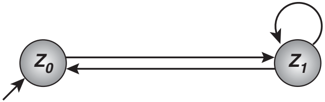


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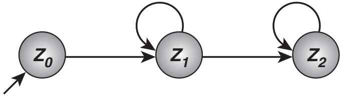
- *2-state+init sequential machines*:
  - one initialising state (visited only once), two working states;

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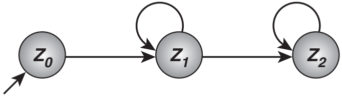


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- *2-state+init alternating machines*:
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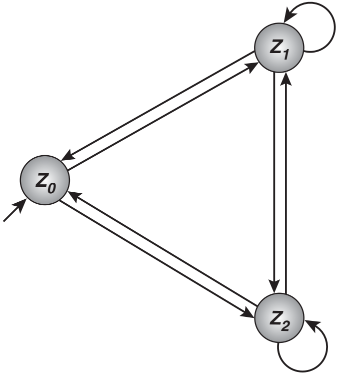
  - arbitrary transitions between any states are possible.



## Generalisations:

- *k-state+init sequential machines*:
  - one initialising state (visited only once), $k$ working states;
  - every search trajectory consists of $1+k$ phases.

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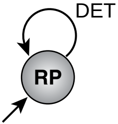
# Modelling SLS Methods Using GLSMs

## Uninformed Picking and Uninformed Random Walk





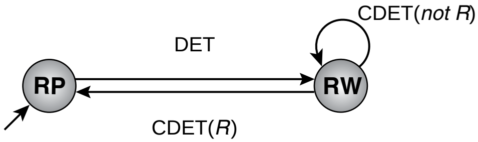
**procedure** *step-RP*$(\pi, s)$
  **input:** *problem instance* $\pi \in \Pi$,
    *candidate solution* $s \in S(\pi)$
  **output:** *candidate solution* $s \in S(\pi)$

  $s' := selectRandom(S)$;
  **return** $s'$
**end** *step-RP*

**procedure** *step-RW*$(\pi, s)$
  **input:** *problem instance* $\pi \in \Pi$,
    *candidate solution* $s \in S(\pi)$
  **output:** *candidate solution* $s \in S(\pi)$

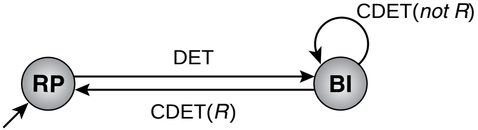
  $s' := selectRandom(N(s))$;
  **return** $s'$
**end** *step-RW*

## Uninformed Random Walk with Random Restart




$R$ = restart predicate, *e.g.*, countm($k$)

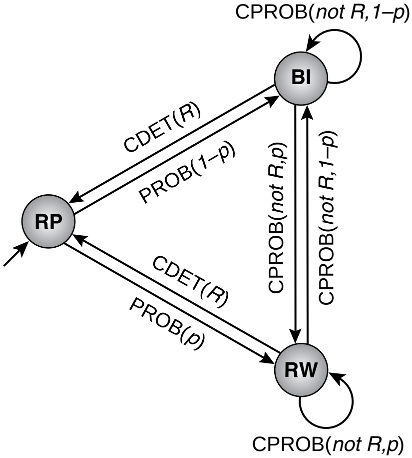
## Iterative Best Improvement with Random Restart



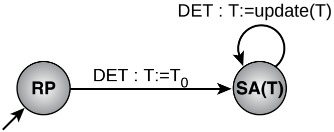

**procedure** *step-BI*($\pi, s$)
  **input:** *problem instance* $\pi \in \Pi$, *candidate solution* $s \in S(\pi)$
  **output:** *candidate solution* $s \in S(\pi)$

  $g^* := \min\{g(s') \mid s' \in N(s)\}$;
  $s' := selectRandom(\{s' \in N(s) \mid g(s') = g^*\})$;
  **return** $s'$
**end** *step-BI*

# Randomised Iterative Best Improvement with Random Restart



## Simulated Annealing




- Note the use of transition actions and memory for temperature $T$.
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- The initial temperature $T_0$ and function *update* implement the annealing schedule.

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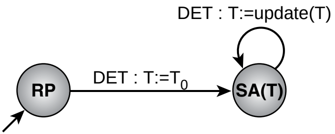


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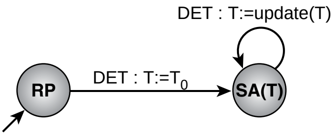


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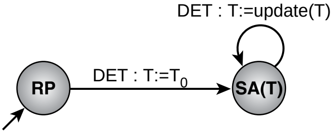


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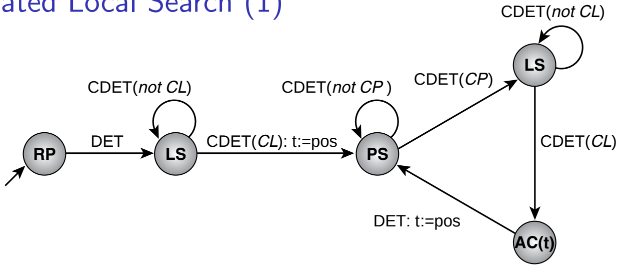


- The acceptance criterion is modelled as a state type, since it affects the search position.
- Note the use of transition actions for memorising the current candidate solution (pos) at the end of each local search phase.
- Condition predicates *CP* and *CL* determine the end of perturbation and local search phases, respectively; in many ILS algorithms, $CL :=$ lmin.

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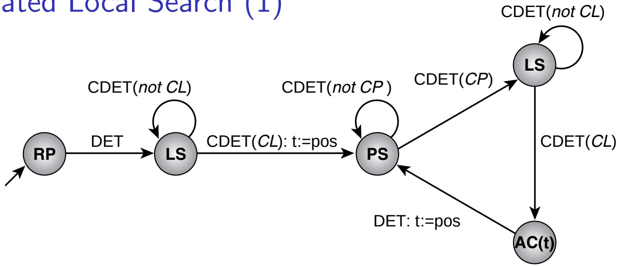


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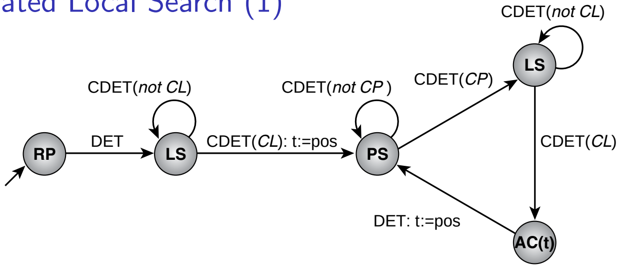


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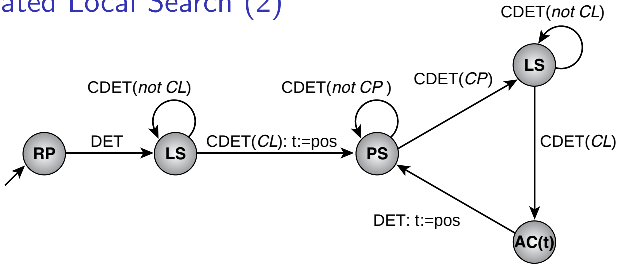


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  **output:** *candidate solution* $s \in S(\pi)$
  **if** $C(\pi, s, t)$ **then**
    **return** $s$
  **else**
    **return** $t$
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**end** *step-AC*

## Ant Colony Optimisation (1)

- General approach for modelling population-based SLS methods, such as ACO, as GLSMs:

  Define search positions as *sets of candidate solutions*; search steps manipulate some or all elements of these sets.

  *Example:* In this view, Iterative Improvement (II) applied to a population *sp* in each step performs one II step on each candidate solution from *sp* that is not already a local minimum.

  (Alternative approaches exist.)

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- General approach for modelling population-based SLS methods, such as ACO, as GLSMs:

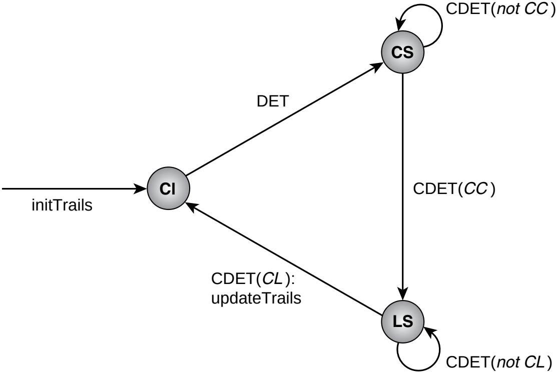
  Define search positions as *sets of candidate solutions*; search steps manipulate some or all elements of these sets.

  *Example:* In this view, Iterative Improvement (II) applied to a population *sp* in each step performs one II step on each candidate solution from *sp* that is not already a local minimum.

  (Alternative approaches exist.)

- Pheromone levels are represented by memory states and are initialised and updated by means of transition actions.

# Ant Colony Optimisation (2)




- The condition predicate *CC* determines the end of the construction phase.
- The condition predicate *CL* determines the end of the local search phase; in many ACO algorithms, *CL* := lmin.

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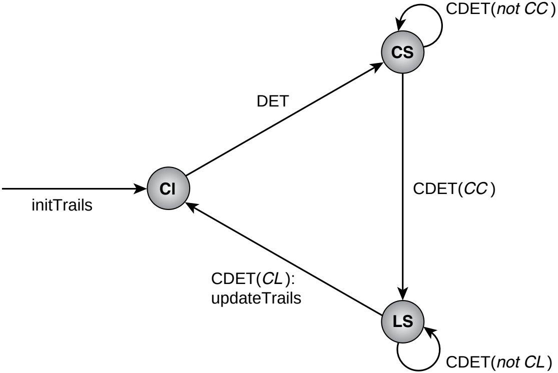


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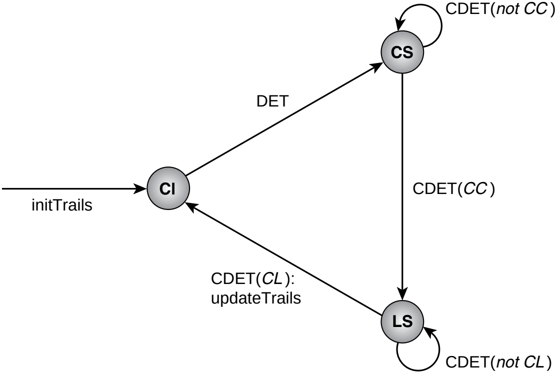


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# Extensions of the Basic GLSM Model

The basic GLSM model can be generalised and extended in various rather straightforward ways, such as:

- Co-operative GLSM models
- Learning GLSM models
- Evolutionary GLSM models
- Continuous GLSM models

**Note:** So far, these extensions remain mostly unexplored — lots of opportunities for interesting research!

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## Co-operative GLSM models

- **Key idea:** Apply multiple GLSMs simultaneously to the same problem instance
- Naturally captures population-based SLS approaches.
- *Homogeneous co-operative GLSM models:*
  Population of identical GLSMs; equivalent to performing multiple independent runs of the respective SLS method.
- *Heterogenous co-operative GLSM models:*
  Population of different GLSMs; model *algorithm portfolios*.

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- GLSMs in population exchange information about their search trajectories, *e.g.*, via message passing or blackboard mechanism.
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- **Key idea:** In a GLSM with probabilistic transitions, let transition probabilities evolve over time to adaptively optimise search control strategy.
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  Optimise control strategy on one problem instance during search process.
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## Evolutionary GLSM models

- **Key idea:** Achieve learning/adaptation in co-operative GLSM models by varying number or type of individual GLSMs over time.
- Distinction between *single*- and *multi-instance learning* as before; similar mechanisms for controlling adaptation process.
- Can easily model, for example, self-optimising portfolios of SLS algorithms.
- Further extensions:
  - support mutation / recombination operations on GLSMs;
  - additionally support learning in individual GLSMs
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- *Note:* Many previously discussed hybrid SLS methods can be extended to continuous optimisation problems and give rise to high-performance algorithms for solving these.
- The main feature of the GLSM model, namely its clear distinction between *lower-level, simple search strategies* and *higher-level search control*, equally applies to continuous SLS algorithms.
- **Key idea:** Model complex continuous SLS methods by using continuous optimisation procedures as subsidiary local search strategies.

**Note:** The GLSM model is well-suited for modelling algorithms for *hybrid combinatorial problems* that involve discrete as well as continuous solution components.

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