CPSC 259: Data Structures and Algorithms for Electrical Engineers

Sorting

Textbook Reference:
Thareja first edition: Chapter 14: Pages 586-606
Thareja second edition: Chapter 14: Pages 424-456

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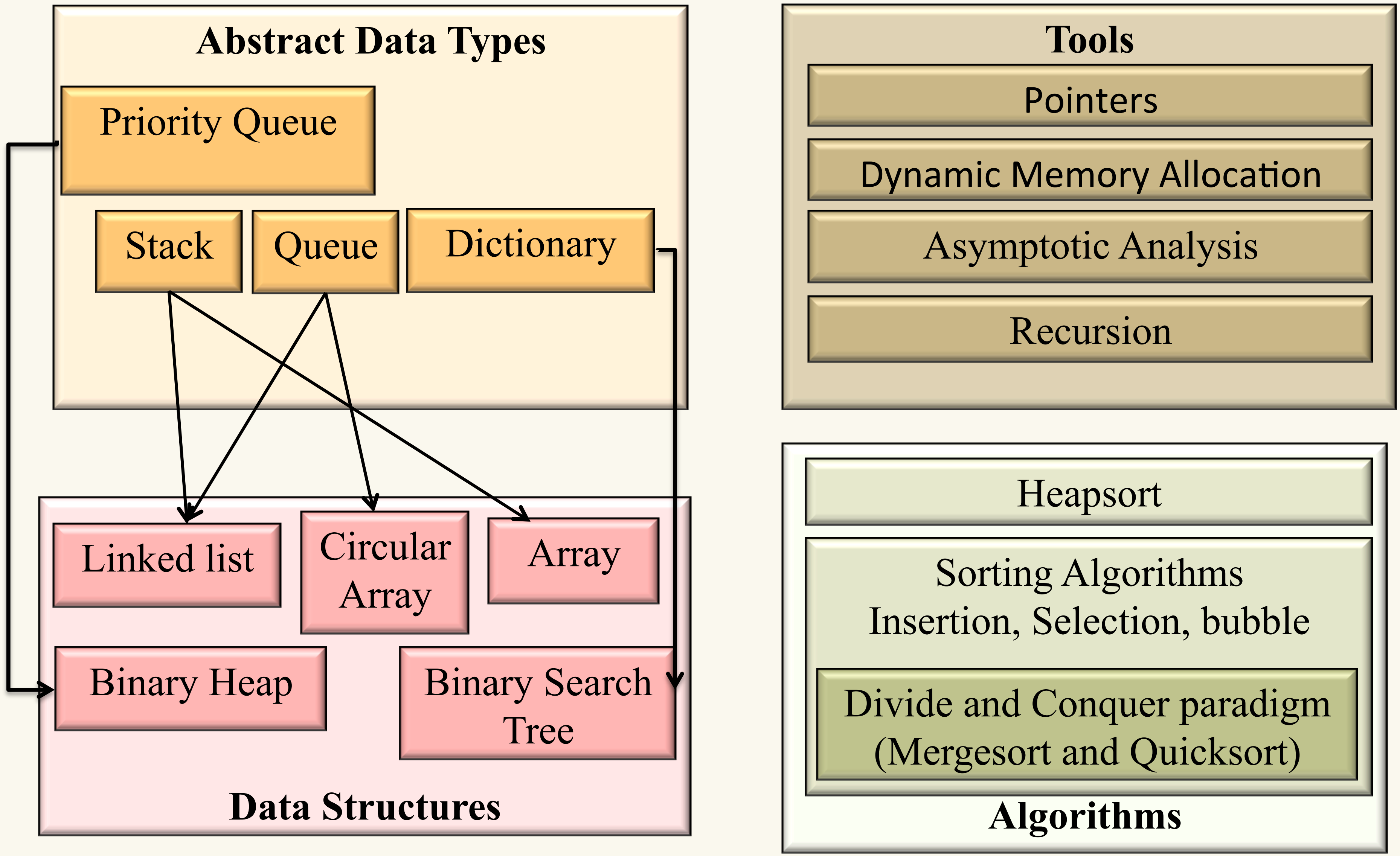
Learning Goals

- Describe and apply various sorting algorithms:
  - Insertion Sort, Selection Sort, Mergesort , Quicksort, Bubble Sort, and Heapsort
- Compare and contrast the tradeoffs of these algorithms.
- State differences in performance for large files versus small files on various sorting algorithms.
- Analyze the complexity of these sorting algorithms.
- Manipulate data using various sorting algorithms (irrespective of any implementation).

CPSC 259 Journey

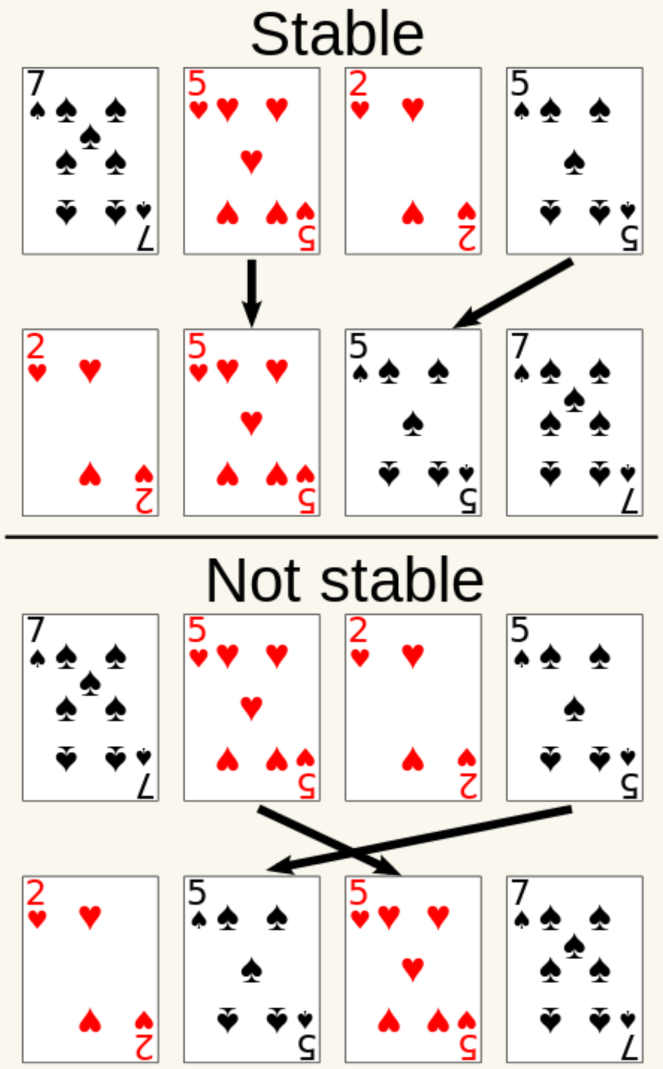


Categorizing Sorting Algorithms

- Computational complexity
  - Average case behaviour: Why do we care?
  - Worst/best case behaviour: Why do we care?
- Memory Usage: How much *extra* memory is used?
- Stability: Stable sorting algorithms maintain the relative order of records with equal keys.

Categorizing Sorting Algorithms

- Stability: Stable sorting algorithms maintain the relative order of records with equal keys.



Source: Wikipedia

Stability example

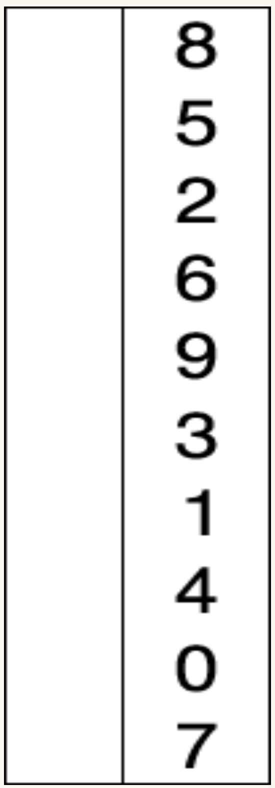
- Stable sorting algorithms maintain the relative order of records with equal keys.

Tyrion Lannister	Sort by l name	Cersei Lannister
Cersei Lannister	Break ties with f name	Jaime Lannister
Daenerys Targaryen		Tyrion Lannister
Jaime Lannister		Daenerys Targaryen

	Sort by f name first	**Sort using a stable sort**
Tyrion Lannister	Cersei Lannister	Cersei Lannister
Cersei Lannister	Daenerys Targaryen	Jaime Lannister
Daenerys Targaryen	Jaime Lannister	Tyrion Lannister
Jaime Lannister	Tyrion Lannister	Daenerys Targaryen

Selection Sort

- Sorts an array by repeatedly finding the smallest element of the unsorted tail region and moving it to the front.



Selection Sort

- Find the smallest and swap it with the first element

5 9 17 11 12

- Find the next smallest. It is already in the correct place

5 9 17 11 12

- Find the next smallest and swap it with first element of unsorted portion

5 9 11 17 12

- Repeat

5 9 11 12 17

- When the unsorted portion is of length 1, we are done

5 9 11 12 17

Selection Sort

```
/*
 Purpose: Find the position of the minimum value
          in part of an array
 Param:   data - integer array to be sorted
          from - starting index
          to   - ending index
 returns  - index of minimum value between from and to
 */

int min_position(int data[], int from, int to)
{
    int min_pos = from;
    int i;
    for (i = from + 1; i <= to; i++)
        if (data[i] < data[min_pos])
           min_pos = i;
    return min_pos;
}
```

Selection Sort

```
/*
 Purpose: sorts elements of an array of integers using
          selection sort
 Param:  data - integer array to be sorted
         size - size of the array
 */

void selection_sort(int data[], int size)
{
    int next; // The next position to be set to minimum
    for (next = 0; next < size - 1; next++)
    {
        int min_pos = min_position(data, next, size-1);
        if (min_pos != next)
            swap(&data[min_pos], &data[next]);
    }
}
```

In-Class Exercise

- Write out all the steps that selection sort takes to sort the following sequence:

91	5	11	90	6	16	31	88

In-Class Exercise

- Write out all of the steps that selection sort takes to sort the following sequence:

91	5	11	90	6	16	31	88
5	91	11	90	6	16	31	88
5	6	11	90	91	16	31	88
5	6	11	90	91	16	31	88
5	6	11	16	91	90	31	88
5	6	11	16	31	90	91	88
5	6	11	16	31	88	91	90
5	6	11	16	31	88	90	91

Clicker Question

- What is the time complexity of selection sort in the best and worst case.
- *A: $O(n^2)$, $O(n^2)$*
- *B: $O(n)$, $O(n^2)$*
- *C: $O(n \lg n)$, $O(n \lg n)$*
- *D: $O(n \lg n)$, $O(n^2)$*
- *E: $O(n)$, $O(n \lg n)$*

Clicker Question (answer)

- What is the time complexity of selection sort in the best and worst case.
- *A: $O(n^2)$, $O(n^2)$*
- *B: $O(n)$, $O(n^2)$*
- *C: $O(n \lg n)$, $O(n \lg n)$*
- *D: $O(n \lg n)$, $O(n^2)$*
- *E: $O(n)$, $O(n \lg n)$*

Clicker Question

- Is selection sort stable?
- *A: Yes*
- *B: No*
- *C: I don't know*

Clicker Question

- Is selection sort stable?
- *A: Yes*
- *B: No, but it is possible to make selection sort stable*
- *C: I don't know*

90	5	11	90	6	16	2	88

2	5	11	90	6	16	90	88

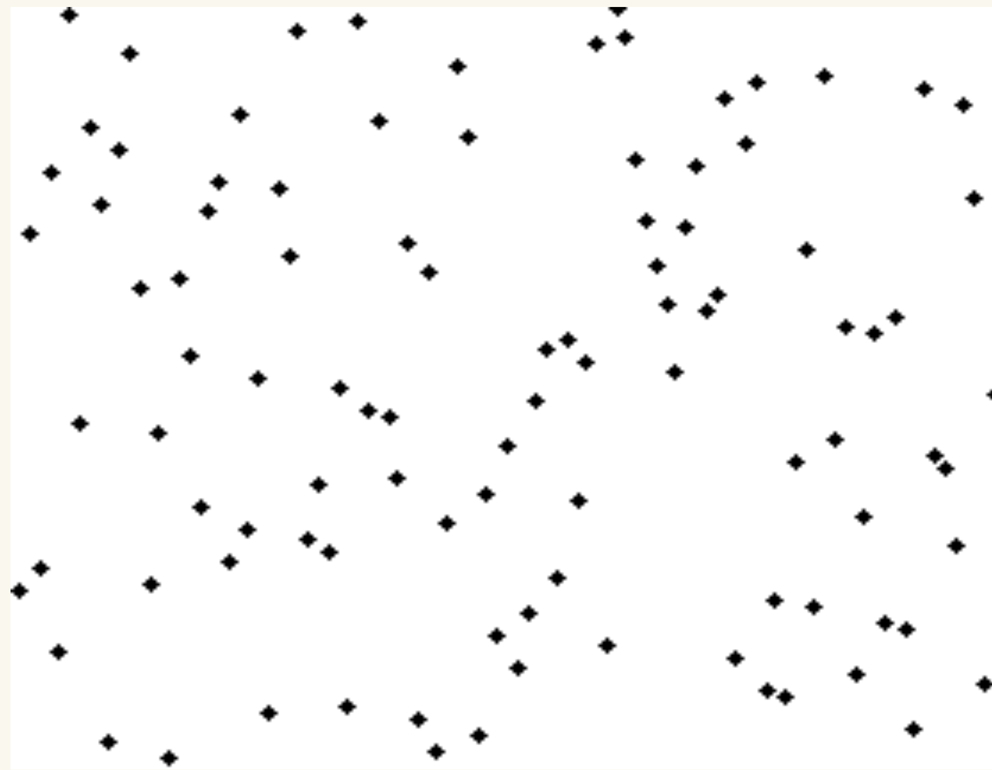
When is the Selection Sort algorithm used?

- One advantage of selection sort is that it requires only *O(n)* write operations. If we have a system where write operations are extremely expensive and read operations are not, then selection sort could be ideal. One such scenario would be if we are sorting a file in-place on flash memory or an external hard drive.

Name	Best	Average	Worst	Stable	Memory
Selection Sort	$O(n^2)$	$O(n^2)$	$O(n^2)$	challenging	O(1)

Insertion Sort

- Given a list, take the current element and insert it at the appropriate position of the list, adjusting the list every time you insert



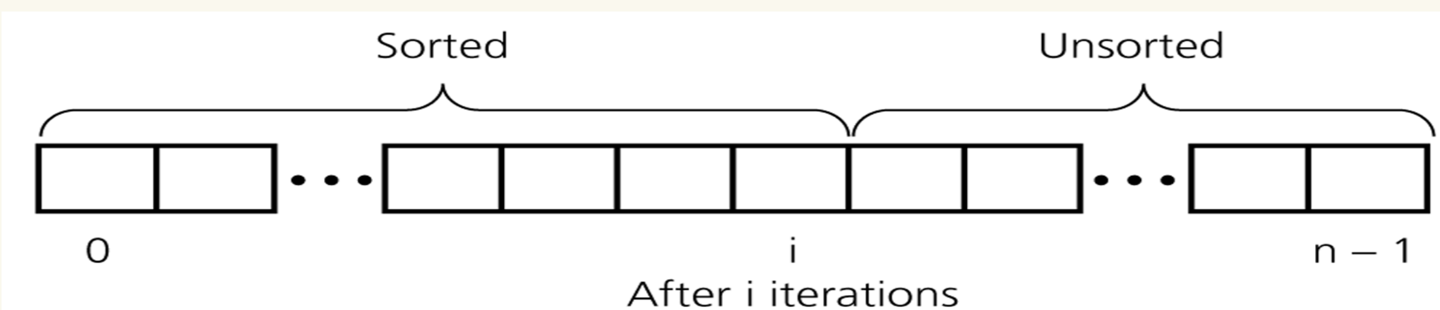
Insertion Sort

- while some elements unsorted:
  - Using linear search, find the location in the sorted portion where the 1st element of the unsorted portion should be inserted
  - Move all the elements after the insertion location up one position to make space for the new element

45

38	45	60	66	79	47	13	74	36	21	94	22	57	16	29	81

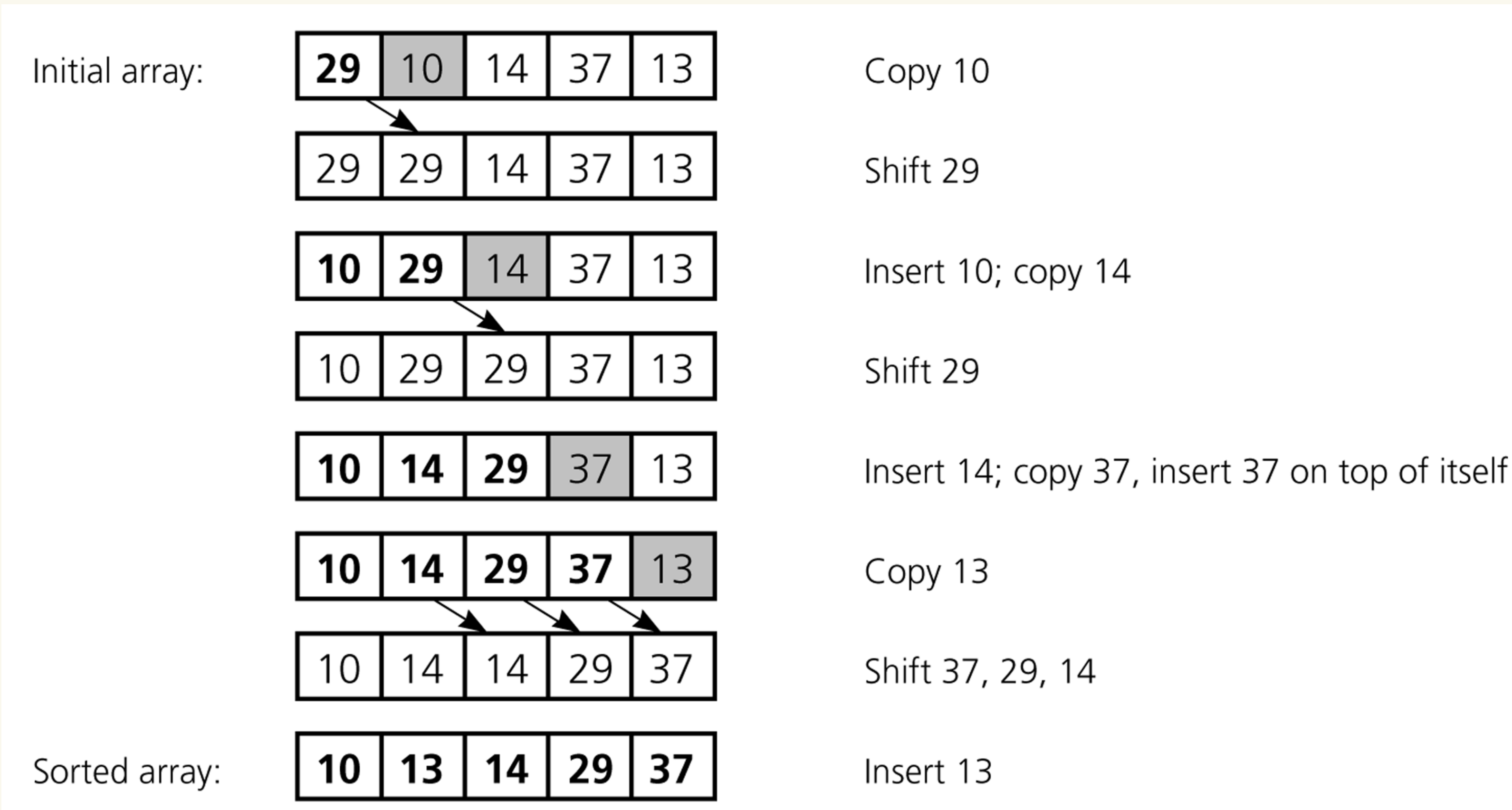
the fourth iteration of this loop is shown here



In-class exercise

- Write out all of the steps that insertion sort takes to sort the following sequence:

29 10 14 37 13



Insertion Sort

```
/*
 Purpose: sorts elements of an array of integers using
insertion sort

 Param:  data   - integer array to be sorted
        length - size of the array
 */

void insertion_sort(int data[], int length){
  for (int i = 1; i < length; i++){
    int val = data [i];
    int newIndex = bSearch(data, val, 0, i);
    for (int j = i; j > newIndex; j--)
      data [j] = data [j-1];

    data [newIndex] = val;
  }
}
```

Clicker question

- What is the time complexity of Insertion Sort in the best and worst case, assuming linear search is used.
- *A*: $O(n^2)$, $O(n^2)$
- *B*: $O(n)$, $O(n^2)$
- *C*: $O(n \lg n)$, $O(n \lg n)$
- *D*: $O(n \lg n)$, $O(n^2)$
- *E*: $O(n)$, $O(n \lg n)$

Clicker question (answer)

- What is the time complexity of Insertion Sort in the best and worst case, assuming linear search is used.

a1	a2	a3	a4	a5

- $$\sum_{i=1}^{n} 1 = n \in O(n)$$

B: $O(n)$, $O(n^2)$

- Best case

$$\sum_{i=1}^{n} i = n(n+1)/2 \in O(n^2)$$

- Worst case

$$\sum_{i=1}^{n} i/2 = n(n+1)/4 \in O(n^2)$$

- Average case

Clicker question

- What is the time complexity of Insertion Sort in the best and worst case, assuming binary search is used.
- *A*: $O(n^2)$, $O(n^2)$
- *B*: $O(n)$, $O(n^2)$
- *C*: $O(n \lg n)$, $O(n \lg n)$
- *D*: $O(n \lg n)$, $O(n^2)$
- *E*: $O(n)$, $O(n \lg n)$

Clicker question

- What is the time complexity of Insertion Sort in the best and worst case, assuming binary search is used.
- *A: $O(n^2)$, $O(n^2)$*
- *B: $O(n)$, $O(n^2)$*
- *C: $O(n \lg n)$, $O(n \lg n)$*
- *D: $O(n \lg n)$, $O(n^2)$*
- *E: $O(n)$, $O(n \lg n)$*

Clicker Question

- Suppose we are sorting an array of ten integers using a sorting algorithm. After four iterations of the algorithm's main loop, the array elements are ordered as shown here:

  1 2 3 4 5 0 6 7 8 9

A. The algorithm might be either selection sort or insertion sort.

B. The algorithm might be selection sort, but could not be insertion sort.

C. The algorithm might be insertion sort, but could not be selection sort.

D. The algorithm is neither selection sort nor insertion sort.

Clicker Question (answer)

- Suppose we are sorting an array of ten integers using a sorting algorithm. After four iterations of the algorithm's main loop, the array elements are ordered as shown here:

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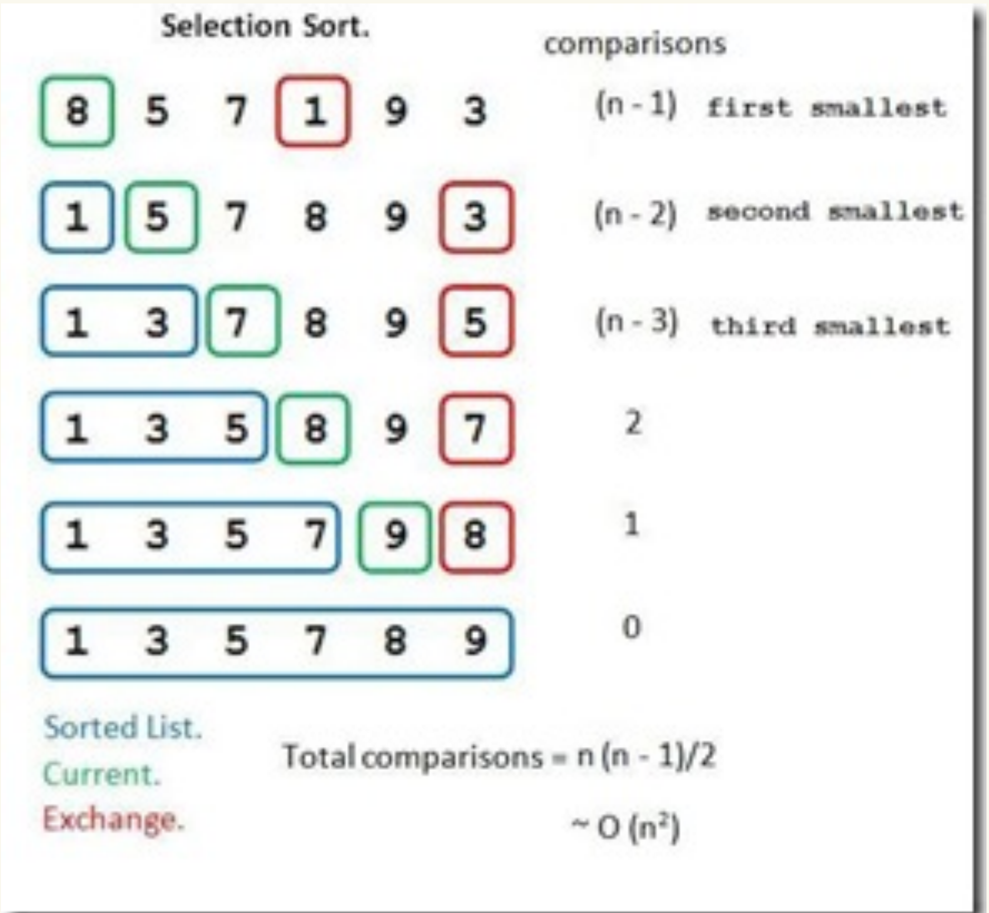
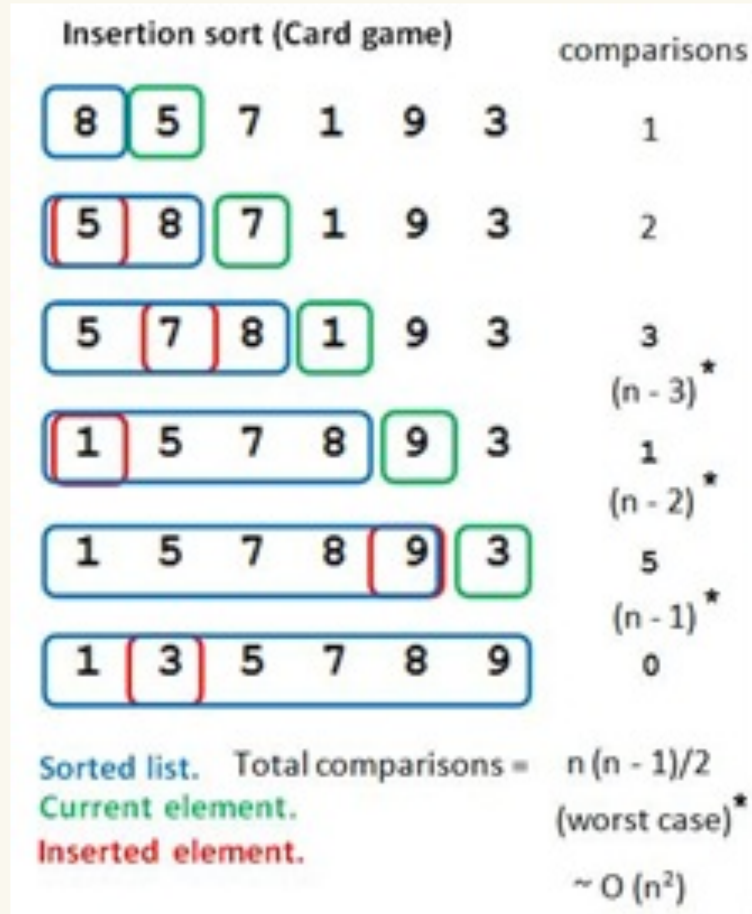
D. The algorithm is neither selection sort nor insertion sort.

Selection Sort vs. Insertion Sort





Source

When is the Insertion Sort algorithm used?

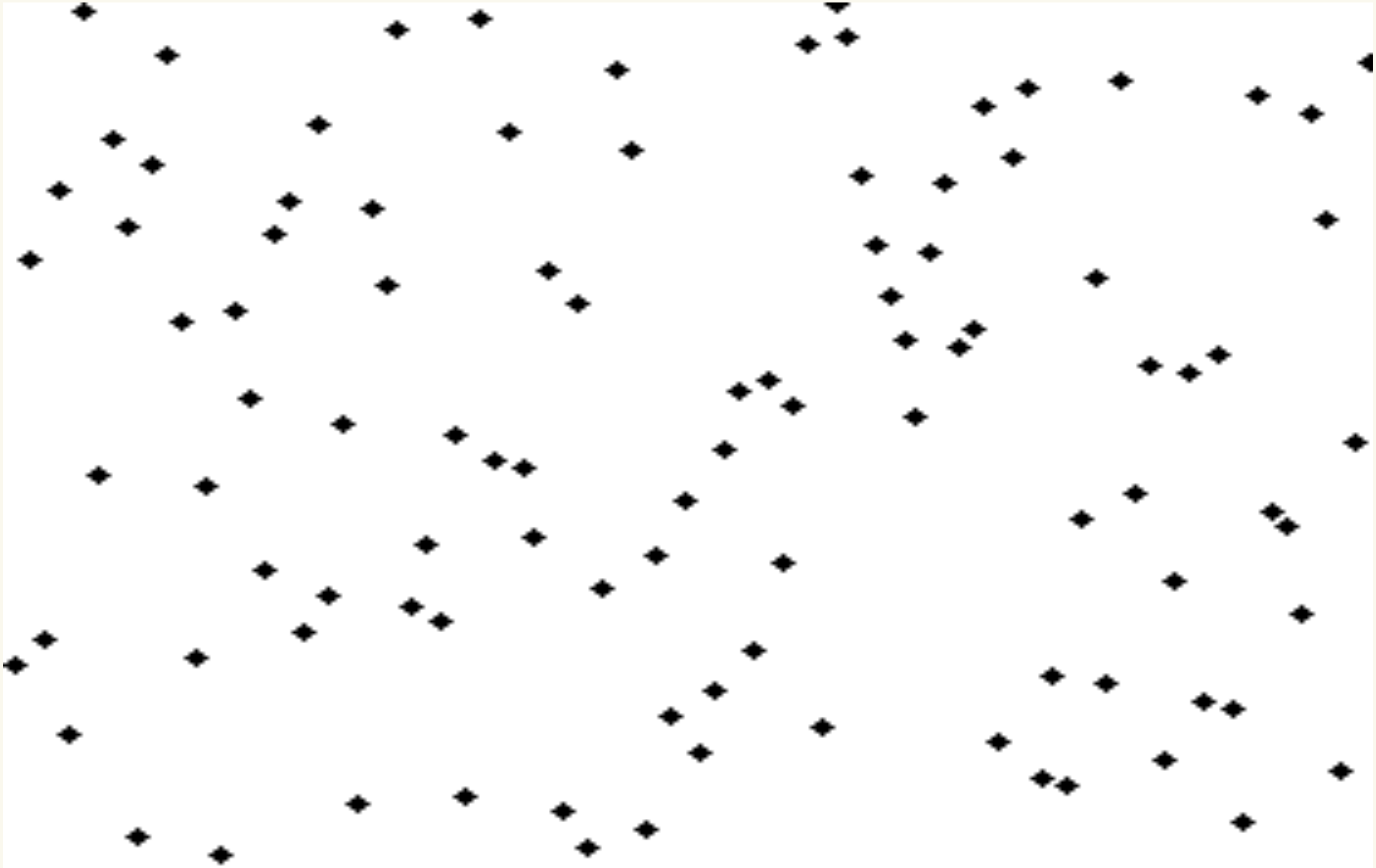
- Insertion Sort is the algorithm of choice either when the data is nearly sorted (because it is adaptive) or when the problem size is small (because it has low overhead).

Name	Best	Average	Worst	Stable	Memory
Selection Sort	$O(n^2)$	$O(n^2)$	$O(n^2)$	challenging	$O(1)$
Insertion Sort	$O(n)$	$O(n^2)$	$O(n^2)$	Yes	$O(1)$

Mergesort

- Mergesort is an example of a divide-and-conquer algorithm that recursively splits the problem into branches, and later combines them to form the solution.
- **Key Steps in Mergesort**:
  1. Split the array into two halves.
  2. Recursively sort each half.
  3. Merge the two (sorted) halves together to produce a bigger, sorted array.
     - Note: The time to merge two sorted sub-arrays of sizes m and n is linear: $O(m + n)$.

Mergesort



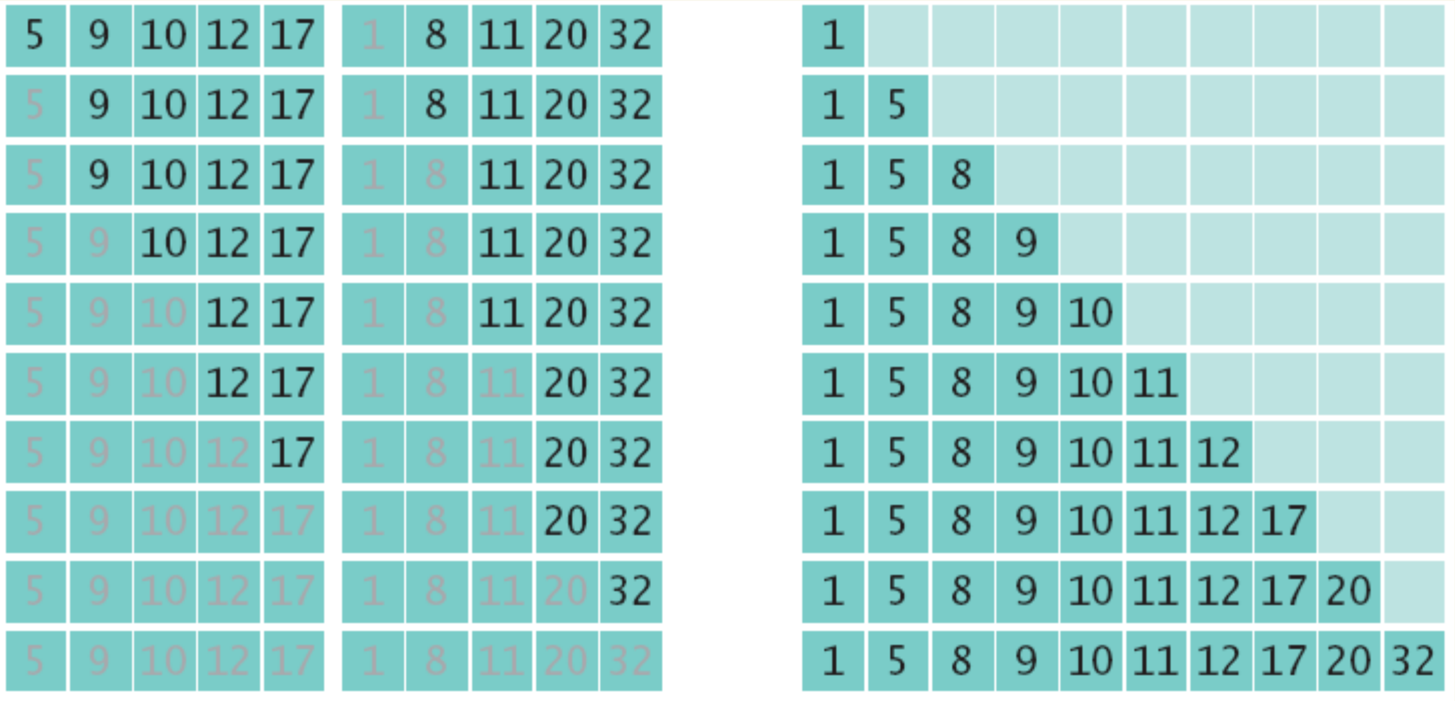
MergeSort

```
void msort(int x[], int lo, int hi, int tmp[]) {
  if (lo >= hi) return;
  int mid = (lo+hi)/2;
  msort(x, lo, mid, tmp);
  msort(x, mid+1, hi, tmp);
  merge(x, lo, mid, hi, tmp);
}

void mergeSort(int x[], int n) {
     /* temp. space */
    int * tmp = (int *) malloc(n * sizeof(int));

    msort(x, 0, n-1, tmp);
    free(tmp);
}
```

Merging two sorted arrays

- Divide an array in half and sort each half

- Merge the two sorted arrays into a single sorted array



Merge by Jon Bentley

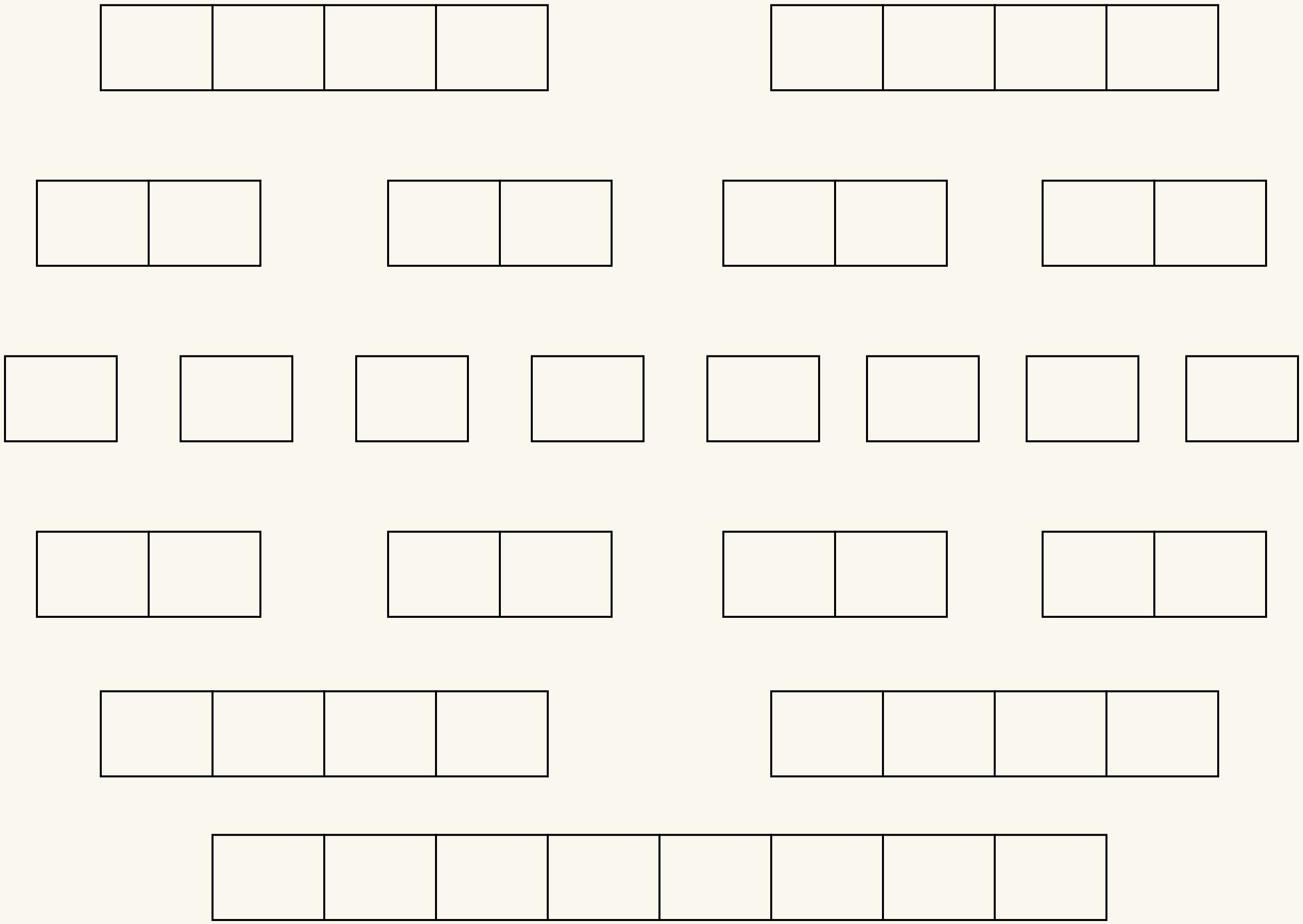
```
/*
 Purpose:  Merges two adjacent ranges in an array

 param     x    - the array with the elements to merge
          low   - the start of the first range
          mid   - the end of the first range
          hi    - end of the second range
          tmp[]- temp memory used for sorting
 */
void merge(int x[], int lo, int mid, int hi, int tmp[]) {
    int a = lo, b = mid+1, k;
    for( k = lo; k <= hi; k++ )
        if( a <= mid && ( b > hi || x[a] < x[b] ) )
            tmp[k] = x[a++];  /* store x[a] then a++ */
        else
            tmp[k] = x[b++]; /* store x[b] then b++ */
    for( k = lo; k <= hi; k++ )
        x[k] = tmp[k];
}
```

Elegant & brilliant…
but not how I'd write it.

In-class exercise

- Write out all the steps that MergeSort takes to sort the following sequence:

3	-4	7	5	9	6	2	1



In-class exercise

- Write out all the steps that Mergesort takes to sort the following sequence:

3	-4	7	5	9	6	2	1

3	-4	7	5

9	6	2	1

3	-4

7	5

9	6

2	1

*

3	-4	7	5	9	6	2	1

-4	3

5	7

6	9

1	2

**

-4	3	5	7

1	2	6	9

-4	1	2	3	5	6	7	9

```
merge( x, 0, 0, 1, tmp );/* step * in previous slide*/
```

x:

3	-4	7	5	9	6	2	1

tmp:

-4	3

x:

-4	3	7	5	9	6	2	1

```
merge( x, 4, 5, 7, tmp ); /* step ** in previous slide*/
```

x:

-4	3	5	7	6	9	1	2

tmp:

				1	2	6	9

x:

-4	3	5	7	1	2	6	9

```
merge( x, 0, 3, 7, tmp );  /* the final step */
```

Clicker question

- Mergesort makes two recursive calls. Which statement is true after these recursive calls finish, but before the merge step?

  A. The array elements form a heap.

  B. Elements in each half of the array are sorted amongst themselves.

  C. Elements in the first half of the array are less than or equal to elements in the second half of the array.

  D. None of the above

Clicker question

- Mergesort makes two recursive calls. Which statement is true after these recursive calls finish, but before the merge step?
  A. The array elements form a heap.
  B. Elements in each half of the array are sorted amongst themselves.
  C. Elements in the first half of the array are less than or equal to elements in the second half of the array.
  D. None of the above

Clicker Question

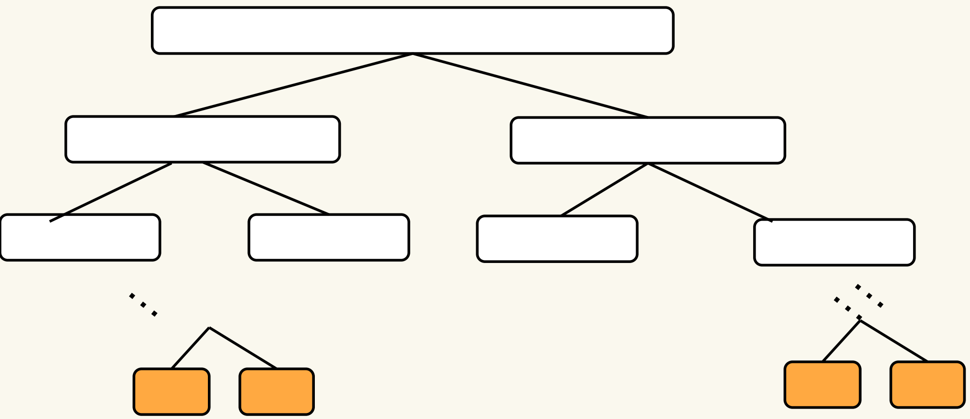
- Is Mergesort stable?
- A: Yes
- B: No
- C: I don’t know

Clicker Question

- Is Mergesort stable?
- A: Yes
- B: No
- C: I don't know

prefer the "left" of the two sorted sublists on ties

Analyzing the Mergesort Algorithm



O(n) operations at each level

We have lg n levels therefore,

O(n lg n)

depth	# instances	Size of instances	# read/write operations
0	1	n	n $\rightarrow$ n
1	2	n/2	2 * n/2 $\rightarrow$ n
2	4	n/4	4 * n/4 $\rightarrow$ n
…	…	…	
k	2^{k}	$n/2^{k}$	$2^{k} * n/2^{k}$ $\rightarrow$ n
…	…	…	
lg n	$2^{\lg n} \rightarrow n$	$n/2^{\lg n} \rightarrow 1$	$2^{\lg n} * 1$ $\rightarrow$ n

Analyzing the Mergesort Algorithm

$$T(1) <= b \qquad \text{if } n <= 1$$
$$T(n) <= 2T(n/2) + cn \qquad \text{if } n > 1$$

- **Analysis**

$T(n) <= 2T(n/2) + cn$

$<= 2(2T(n/4) + c(n/2)) + cn$ (substitution)

$= 4T(n/4) + cn + cn$

$<= 4(2T(n/8) + c(n/4)) + cn + cn$ (substitution)

$= 8T(n/8) + cn + cn + cn$

$<= 2^kT(n/2^k) + kcn$ (extrapolating $1 < k \leq n$)

Analyzing the Mergesort Algorithm

```
T(1) <= b                    if n <= 1
T(n) <= 2T(n/2) + cn         if n > 1
```

- To make the analysis easier, let's say we want to analyze the algorithm for $n=2^m$. Since n can still be arbitrary large, there is no loss of generality.

$T(n) \le 2^m T(n/2^m) + mcn$ (extrapolating for m)

$\le nT(1) + cn \lg n$ (for $2^m = n$ or $m = \lg n$)

- $T(n) \in O(n \lg n)$

When is the Mergesort algorithm used?

- External sorting is a term for a class of sorting algorithms that can handle massive amounts of data. External sorting is required when the data being sorted do not fit into the main memory of a computing device (usually RAM) and instead they must reside in the slower external memory (usually a hard drive). Mergesort is suitable for external sorting.
- Mergesort is also highly parallelizable

Name	Best	Average	Worst	Stability	Memory
Selection Sort	$O(n^2)$	$O(n^2)$	$O(n^2)$	challenging	$O(1)$
Insertion Sort	$O(n)$	$O(n^2)$	$O(n^2)$	Yes	$O(1)$
Mergesort	$O(n \lg n)$	$O(n \lg n)$	$O(n \lg n)$	Yes	$O(n)$

Quicksort

- In practice, one of the fastest sorting algorithms is Quicksort, developed in 1961 by Hoare.
- Comparison-based: examines elements by comparing them to other elements
- Divide-and-conquer: divides into “halves” (that may be very unequal) and recursively sorts

Quicksort algorithm

- Pick a pivot
- Reorder the list such that all elements < pivot are on the left, while all elements >= pivot are on the right
- Recursively sort each side

Are we missing a base case?

Partitioning

- The act of splitting up an array according to the pivot is called partitioning
- Consider the following: $\underbrace{-4\quad 1\quad -3\quad 2}_{\texttt{left partition}}\quad \underset{\textbf{pivot}}{3}\quad \underbrace{5\quad 4\quad 7}_{\texttt{right partition}}$

- This algorithm will terminate -- each iteration places at least one element, the pivot, in its final spot

- Two cool facts about partitioning
  - Runs in linear time with no extra memory
  - Reduces problem size beautifully

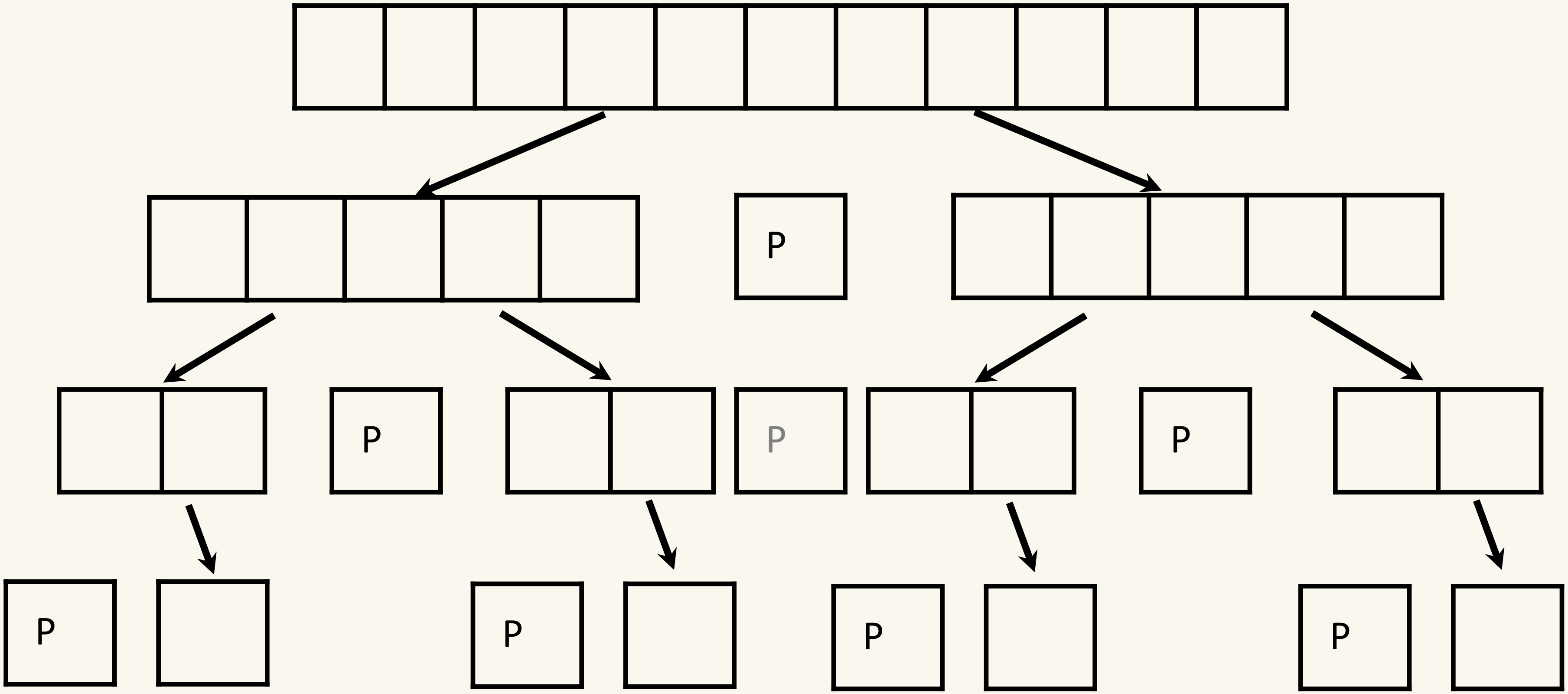
Quicksort example

- http://visualgo.net/sorting.html
  - Initialize array with
    - 25, 44, 38, 5, 47, 15, 36, 26, 27, 2, 46, 4, 19, 50, 48

qSort code

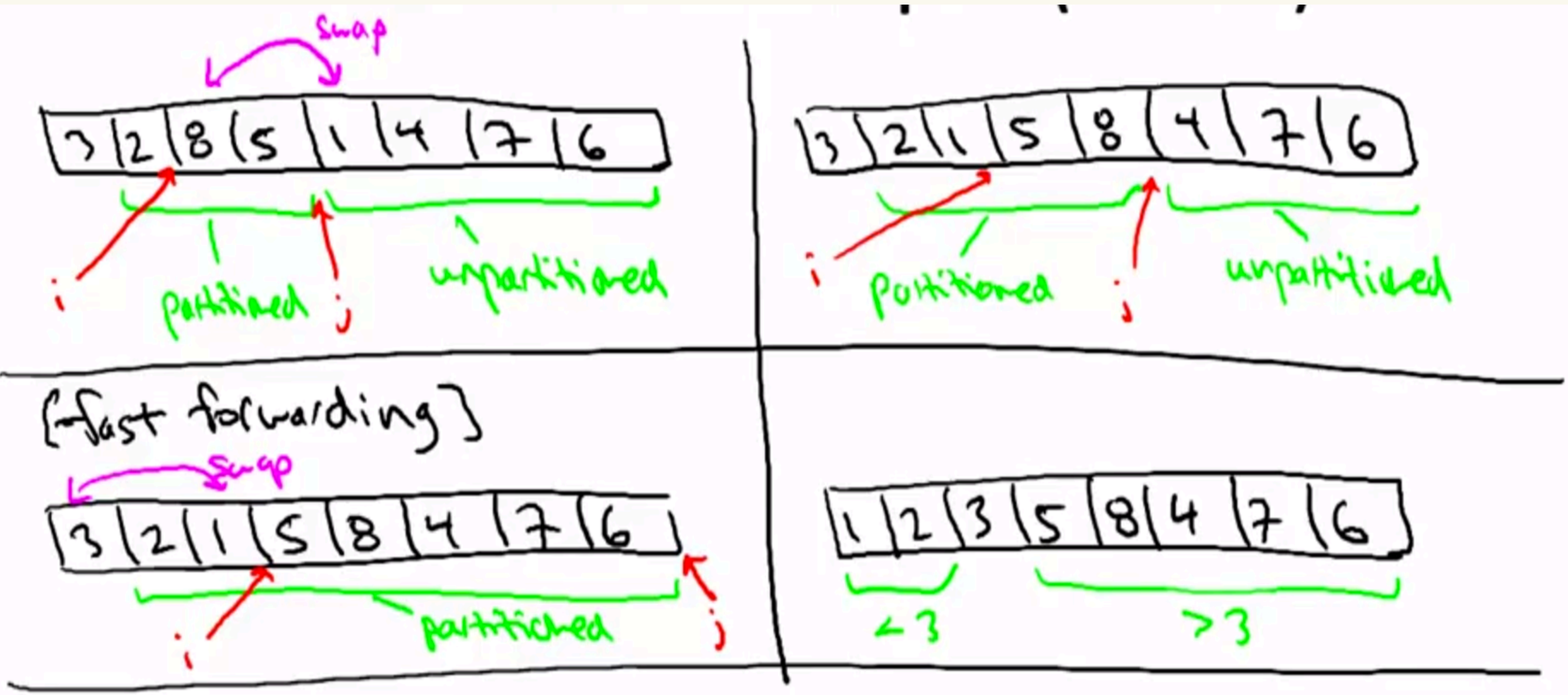
```
/*
 Purpose: sorts elements of an array of integers using Quicksort

 Param:  a  - integer array to be sorted
         lo - the start of the sequence to be sorted.
         hi - the end of the sequence to be sorted.
 */

void qSort( int a[], int lo, int hi ){
    int pivotElement;

    if(lo < hi){
        pivotElement = pivot(a, lo, hi);
        qSort(a, lo, pivotElement-1);
        qSort(a, pivotElement+1, hi);
    }
}
```

QuickSort Visually



Sorted!

Partitioning example



Partitioning

```
/*
 Purpose: find and return the index of pivot element such that all items
          left of partition are smaller and right of partition are bigger

 Param:   x  - integer array to be sorted
          lo - the start of the sequence to be sorted.
          hi - the end of the sequence to be sorted.
*/
int pivot(int* x, int lo, int hi){
  int j;
  int  i = lo;
  int pivotElement = x[lo];
  for(j = lo+1 ; j <= hi ; j++){
    if(x[j] <= pivotElement){
      i++;
      swap(&x[j], &x[i]);
    }
  }
  swap(&x[i], &x[lo]);
  return i;
}
```

In-class example

- Use QuickSort with the left most array entry selected as the pivot element to sort the following elements. Show all steps.

33	64	65	75	25	66	94	7	10	57
10	25	7	33	64	66	94	65	75	57
7	10	25	33	64	66	94	65	75	57
7	10	25	33	57	64	94	65	75	66
7	10	25	33	57	64	66	65	75	94
7	10	25	33	57	64	65	66	75	94

QuickSort practice question

`2   -4   6   1   5   -3   3   7`

Clicker question

- Here is an array which has just been partitioned by the first step of Quicksort:

  3, 0, 2, 4, 5, 8, 7, 6, 9

Which of these elements could be the pivot?

  - a. 3
  - b. 4
  - c. 5
  - d. 6
  - e. (b) or (c)

Clicker question (answer)

- Here is an array which has just been partitioned by the first step of Quicksort:

  3, 0, 2, 4, 5, 8, 7, 6, 9

Which of these elements could be the pivot?

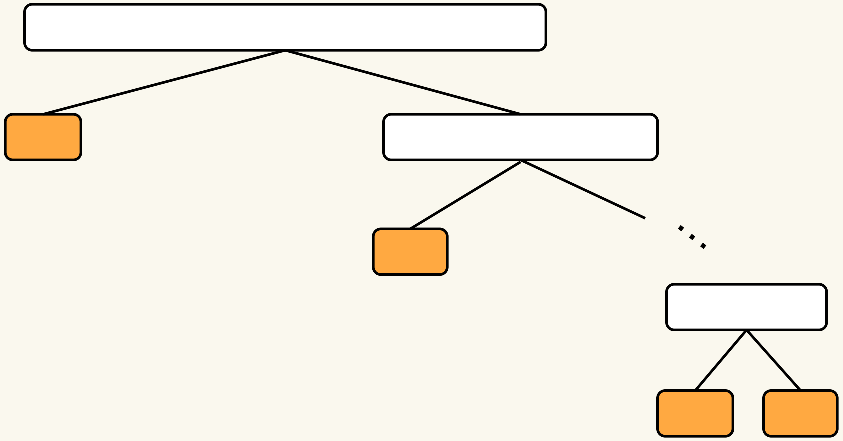
- a. 3
- b. 4
- c. 5
- d. 6
- e. (b) or (c)

Worst-case Running Time

- The worst case for quick-sort occurs when the pivot is the unique minimum or maximum element
- One of $\boldsymbol{L}$ and $\boldsymbol{R}$ has size $\boldsymbol{n} - 1$ and the other has size 0
- The running time is proportional to the sum

$$\boldsymbol{n} + (\boldsymbol{n} - 1) + \ldots + 2 + 1$$

depth	time
0	$\boldsymbol{n}$
1	$\boldsymbol{n} - 1$
…	…
$\boldsymbol{n} - 1$	1



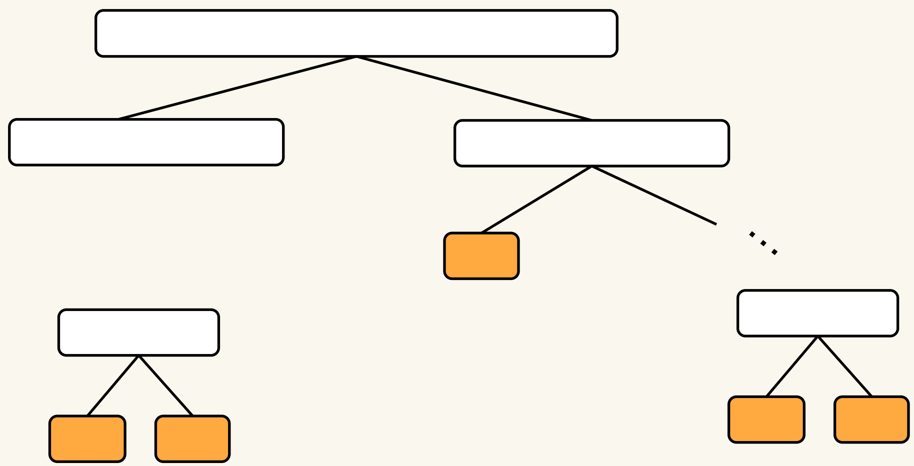
$$\boxed{n + (n-1) + \ldots + 2 + 1 = \frac{n(n+1)}{2} = \frac{n^2}{2} + \frac{n}{2}}$$

Thus, the worst-case running time of quick-sort is $\boldsymbol{O}(\boldsymbol{n}^2)$

Best-case Running Time

- The best case for quick-sort occurs when the pivot is the middle element
- Both of L and G have size $n/2$
  - The height of the quick-sort tree is $O(\lg n)$
  - The amount or work done at the nodes of the same depth is $O(n)$

depth	time
0	n
1	$n/2$
…	…
$Log(n)$	1



  - The running time is $O(n \lg n)$

MergeSort vs. QuickSort

- QuickSort in practice tends to run faster than MergeSort, but its worst-case complexity is $O(n^2)$.
- That worst-case behaviour can usually be avoided by using more clever ways of finding the pivot (not just using the first element).
  – Randomized algorithms can be used to prove that the average case for Quicksort is $O(n \lg n)$

QuickSort: Average Case (Intuition)

- Clearly pivot choice is important
  - It has a direct impact on the performance of the sort
  - Hence, Quicksort is fragile, or at least “attackable”
- So how do we pick a good pivot?
  - Let’s assume that pivot choice is random
  - Half the time the pivot will be in the center half of the array. Thus at worst the split will be n/4 and 3n/4
- The depth of the tree with “good splits” will still O(lg n); therefore running time will be O(n lg n).

Comparison of different sorting algorithms

- Quicksort algorithm is one of the best sorting algorithms and is widely used, and is also highly parallelizable.
- Quicksort is usually done in place with $O(\lg n)$ stack space.

Name	Best	Average	Worst	Stability	Memory
Selection Sort	$O(n^2)$	$O(n^2)$	$O(n^2)$	challenging	$O(1)$
Insertion Sort	$O(n)$	$O(n^2)$	$O(n^2)$	Yes	$O(1)$
Mergesort	$O(n \lg n)$	$O(n \lg n)$	$O(n \lg n)$	Yes	$O(n)$
Quicksort	$O(n \lg n)$	$O(n \lg n)$	$O(n^2)$	Challenging	$O(\lg n)$

Bubble Sort

- **Bubble sort**, works by repeatedly comparing each pair of adjacent items and swapping them if they are in the wrong order.

- See http://visualgo.net/sorting.html for more examples

In-class exercise

Write out all the steps that bubble sort takes to sort the following sequence: (**5 1** 4 2 8)

First Pass:

(**5 1** 4 2 8) → (**1 5** 4 2 8), Swap since 5 > 1
(1 **5 4** 2 8) → (1 **4 5** 2 8), Swap since 5 > 4
(1 4 **5 2** 8) → (1 4 **2 5** 8), Swap since 5 > 2
(1 4 2 **5 8**) → (1 4 2 **5 8**),

Second Pass:

(**1 4** 2 5 8) → (**1 4** 2 5 8)
(1 **4 2** 5 8) → (1 **2 4** 5 8), Swap since 4 > 2
(1 2 **4 5** 8) → (1 2 **4 5** 8)

Third Pass:

(**1 2** 4 5 8) → (**1 2** 4 5 8)
(1 **2 4** 5 8) → (1 **2 4** 5 8)

Fourth Pass: **/* could be avoided */**

(1 **2 4** 5 8) → (1 **2 4** 5 8)

Bubble Sort

- Consider the following implementation for Bubble sort.

```
/*
 Purpose: sorts elements of an array of integers using bubble sort

 Param:  x - integer array to be sorted
 n - size of the array
 */
void bubbleSort(int x[], int n){
  int i, j, flag = 1;    // set flag to 1 to start first pass
  for(i = 1; (i <= n) && flag; i++){
    flag = 0;
    for (j=0; j < (n -1); j++){
      if (x[j+1] < x[j]) {
        swap(&x[j], &x[j+1]);
        flag = 1; // indicates that a swap occurred.
      }
    }
  }
  return;
}
```

Clicker question

- What is the time complexity of Bubble Sort in the best and worst case.
- *A*: $O(n^2)$, $O(n^2)$
- *B*: $O(n)$, $O(n^2)$
- *C*: $O(n \lg n)$, $O(n \lg n)$
- *D*: $O(n \lg n)$, $O(n^2)$
- *E*: $O(n)$, $O(n \lg n)$

Clicker question (answer)

- What is the time complexity of Bubble Sort in the best and worst case.
- *A*: $O(n^2)$, $O(n^2)$
- *B*: $O(n)$, $O(n^2)$
- *C*: $O(n \lg n)$, $O(n \lg n)$
- *D*: $O(n \lg n)$, $O(n^2)$
- *E*: $O(n)$, $O(n \lg n)$

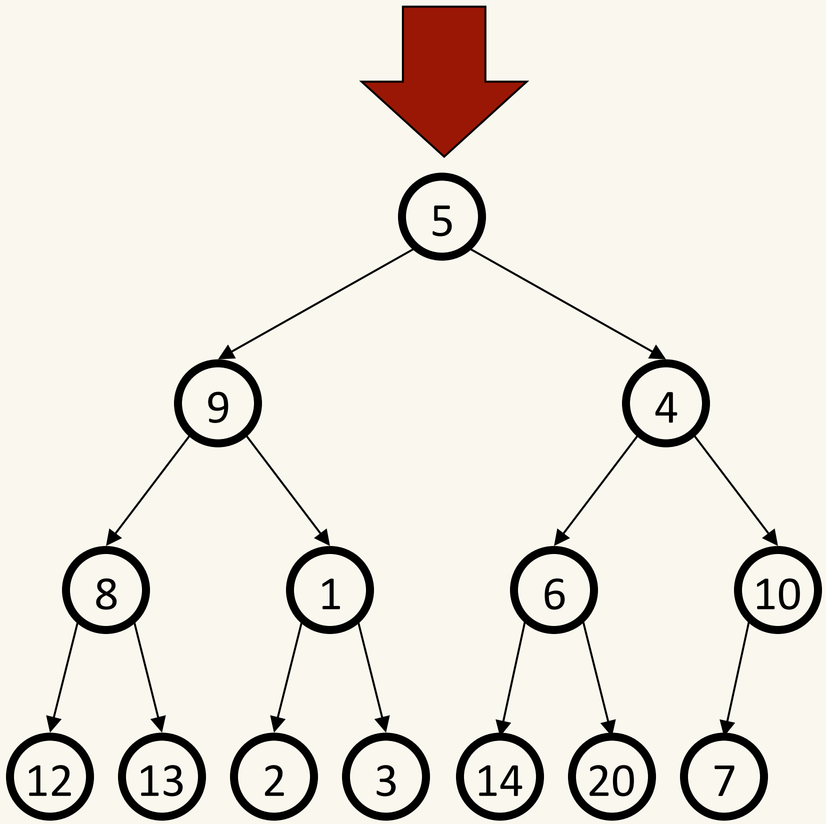
Comparison of different sorting algorithms

- Bubble sort is a simple algorithm that can be used efficiently on a list of any length that is nearly sorted.
- Rarely used in practice.

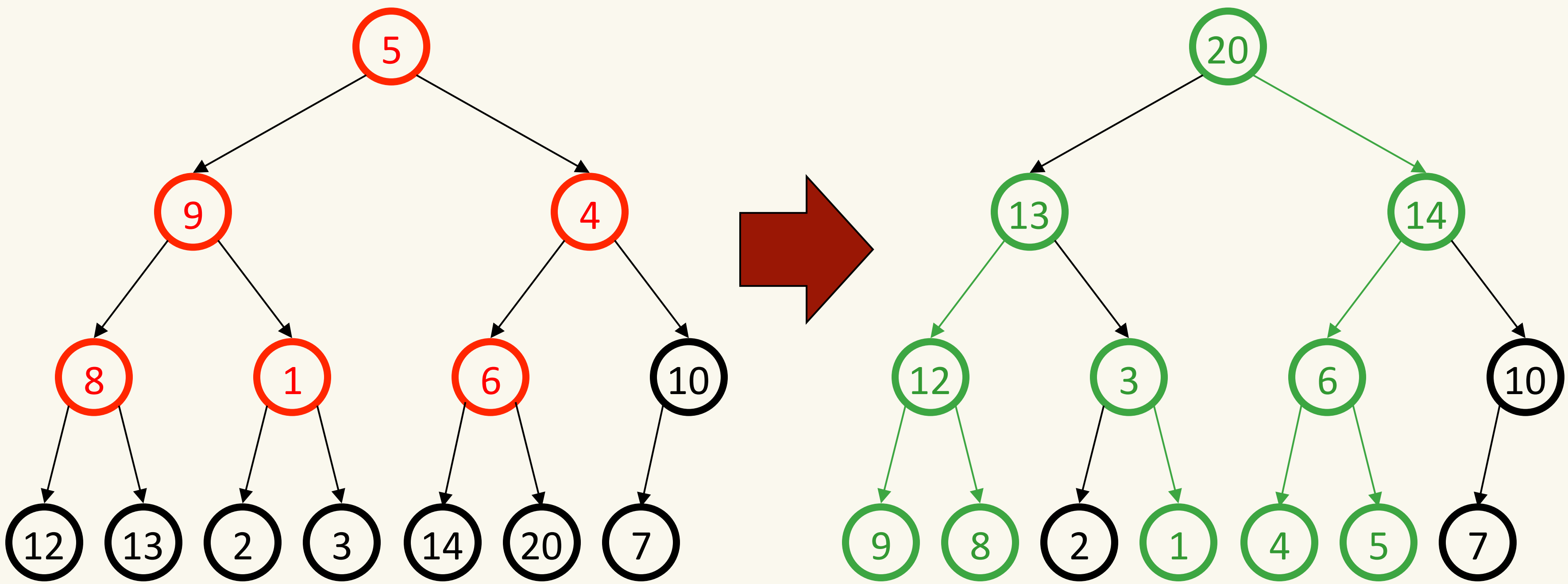
Name	Best	Average	Worst	Stability	Memory
Selection Sort	$O(n^2)$	$O(n^2)$	$O(n^2)$	Challenging	$O(1)$
Insertion Sort	$O(n)$	$O(n^2)$	$O(n^2)$	Yes	$O(1)$
Mergesort	$O(n \lg n)$	$O(n \lg n)$	$O(n \lg n)$	Yes	$O(n)$
Quicksort	$O(n \lg n)$	$O(n \lg n)$	$O(n^2)$	Challenging	$O(\lg n)$
Bubble Sort	$O(n)$	$O(n^2)$	$O(n^2)$	Yes	$O(1)$

Heapsort (revisited)

5	9	4	8	1	6	10	12	13	2	3	14	20	7

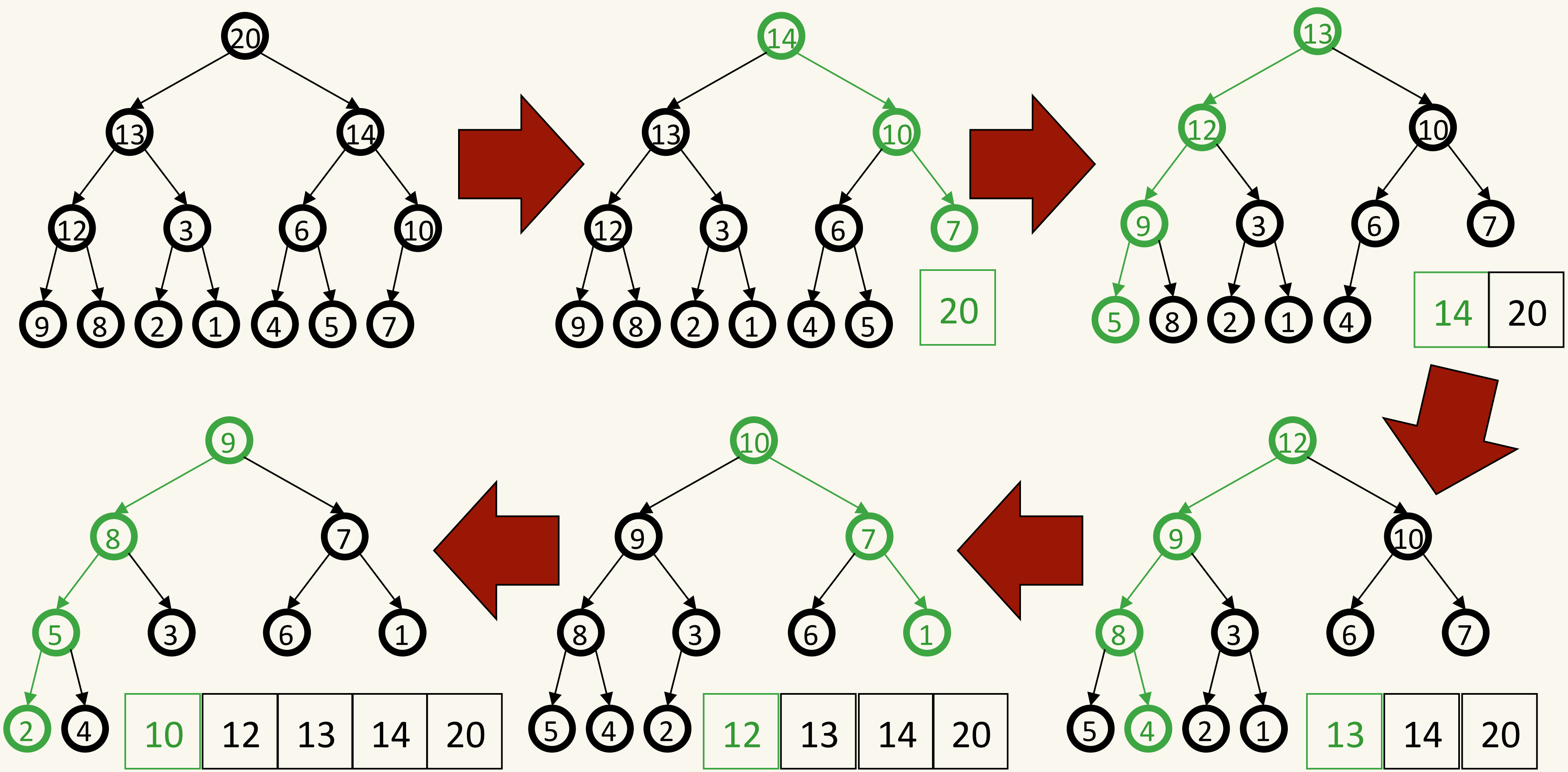


Heapsort (revisited)



Build Heap

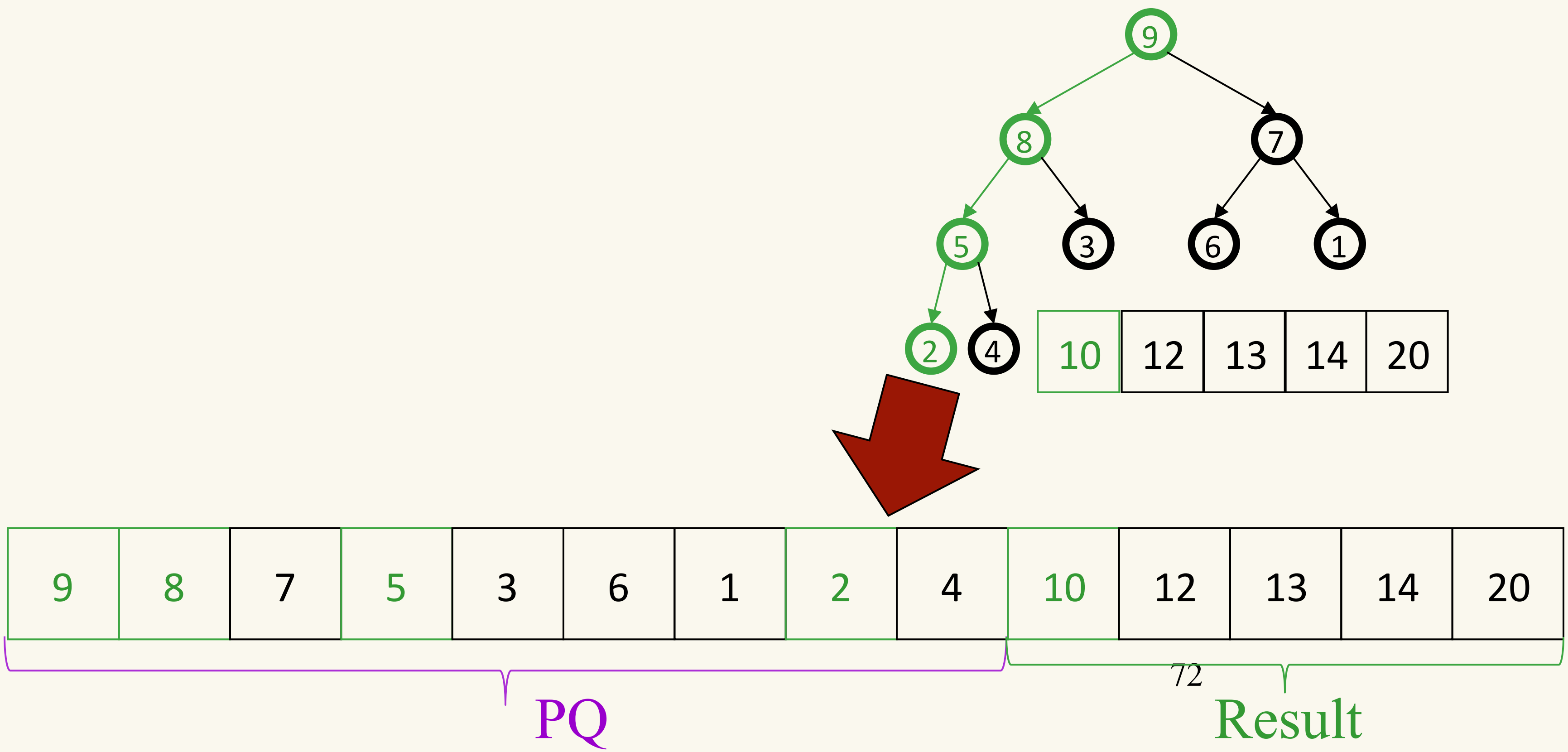
Heapsort (revisited)



Heapsort (revisited)

How long does "build" take? Worst case: O(n) ☺

How long do the deletions take? Worst case: O(n lg n) ☺



Heapsort (revisited)

0	1	2	3	4	5	6	7	8	9	10	11	12	13
5	9	4	8	1	6	10	12	13	2	3	14	20	7

Floyd's Algorithm

20	13	14	12	3	6	10	9	8	2	1	4	5	7

PQ

Heapified Takes only O(n) time!

Heapsort (revisited)

0	1	2	3	4	5	6	7	8	9	10	11	12	13
20	13	14	12	3	6	10	9	8	2	1	4	5	7

PQ

14	13	10	12	3	6	7	9	8	2	1	4	5	20

PQ

13	12	10	9	3	6	7	5	8	2	1	4	14	20

PQ

12	9	10	8	3	6	7	5	4	2	1	13	14	20

PQ

Comparison of different sorting algorithms

- Heapsort can be seen as an efficient version of selection sort, which works by determining the largest (or smallest) element of the list.

Name	Best	Average	Worst	Stability	Memory
Selection Sort	$O(n^2)$	$O(n^2)$	$O(n^2)$	Challenging	$O(1)$
Insertion Sort	$O(n)$	$O(n^2)$	$O(n^2)$	Yes	$O(1)$
Mergesort	$O(n \lg n)$	$O(n \lg n)$	$O(n \lg n)$	Yes	$O(n)$
Quicksort	$O(n \lg n)$	$O(n \lg n)$	$O(n^2)$	Challenging	$O(\lg n)$
Bubble Sort	$O(n)$	$O(n^2)$	$O(n^2)$	Yes	$O(1)$
Heapsort	$O(n \lg n)$	$O(n \lg n)$	$O(n \lg n)$	No	$O(1)$

Average Case Running Time

~7 min

n	Insertion	Heap	Merge	Quick
100,000	26.86s	0.06s	0.03s	0.03s
200,000	108.05s	0.11s	0.08s	0.06s
400,000	437.27s	0.30s	0.17s	0.14s
800,000	?	0.70s	0.34s	0.31s
1,600,000		1.66s	0.72s	0.66s
3,200,000		3.83s	1.52s	1.38s
6,400,000		8.81s	3.08s	2.88s

- How long would it take the insertion sort algorithm to sort 800,000 values

A: 14 minutes B: 28 minutes C: 56 minutes D: other

Average Case Running Time

~7 min

n	Insertion	Heap	Merge	Quick
100,000	26.86s	0.06s	0.03s	0.03s
200,000	108.05s	0.11s	0.08s	0.06s
400,000	437.27s	0.30s	0.17s	0.14s
800,000	?	0.70s	0.34s	0.31s
1,600,000		1.66s	0.72s	0.66s
3,200,000		3.83s	1.52s	1.38s
6,400,000		8.81s	3.08s	2.88s

- How long would it take the insertion sort algorithm to sort 800,000 values
- $T(n) = n^2 \rightarrow \quad T(2n) = 4n^2$ **B: 28 minutes**

Comparison of different sorting algorithms

- Complexity
  - Best case: Insert, Bubble < Quick, Merge, Heap < Select
  - Average case: Quick, Merge, Heap < Insert, Select, Bubble
  - Worst case: Merge, Heap < Quick, Insert, Select, Bubble

- Usually on "real" data: Quick < Merge < Heap < I/S/B
- On very short lists: quadratic sorts may have an advantage (so, some quick/merge implementations "bottom out" to these as base cases)

Comparison of different sorting algorithms

- Stability
  - Easily Made Stable: Insert, Merge, Bubble
  - Challenging to Make Stable: Select, Quick
  - Unstable: Heap

- Memory use:
  - Insert, Select, Heap, Bubble < Quick < Merge

Learning Goals revisited

- Describe and apply various sorting algorithms:
  - Insertion Sort, Selection Sort, Mergesort, Quicksort, Bubble Sort, and Heapsort
- Compare and contrast the tradeoffs of these algorithms.
- State differences in performance for large files versus small files on various sorting algorithms.
- Analyze the complexity of these sorting algorithms.
- Manipulate data using various sorting algorithms (irrespective of any implementation).