Adaptive Multipreconditioning for Domain Decomposition

Nicole Spillane [1]

Multipreconditioning is a technique that allows to simultaneously use several preconditioners within a Krylov subspace solver. It was first introduced in [1] for the conjugate gradient method. The idea is that at each iteration, instead of minimizing the error over one search direction (the preconditioned residual), the error is minimized over an N-dimensional space (spanned by the N preconditioned residuals, where N is the number of preconditioners). Quite naturally, this significantly enlarged search space leads to robust solvers that can converge in a small number of iterations.

Domain decomposition methods are natural candidates for multipreconditioning. Indeed they all share the idea to split the domain into subdomains and then use a **sum of local solves** (one inside each of the subdomains) as a preconditioner. With multipreconditioning, no sum is performed and instead **each local contribution to the preconditioner is kept separate and used to enlarge the search space**. As an illustration, within multipreconditioned Additive Schwarz the search space at a given iteration (with residual r) is spanned by $\{R_i^\top A_i^{-1} R_i\, r\}_{i=1,\dots,N}$ (N-dimensional) instead of $\left(\sum_{i=1}^N R_i^\top A_i^{-1} R_i\right) r$ (unidimensional).

The drawback is of course that each iteration becomes more expensive. For this reason an adaptive multipreconditioned conjugate gradient algorithm was introduced in [3] where only some iterations of the Krylov subspace methods are multipreconditioned. In this talk I will discuss how to choose the adaptivity process and show some numerical results [2] that were obtained in collaboration with C. Bovet, P. Gosselet, and A. Parret-Fréaud on test cases that arise in aircraft engineering.

References

[1] R. Bridson and C. Greif, *A multipreconditioned conjugate gradient algorithm*, SIAM J. Matrix Anal. Appl., 27(4):1056–1068 (electronic), 2006.

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[3] N. Spillane, *An adaptive multipreconditioned conjugate gradient algorithm*, SIAM J. Sci. Comput., 38(3):A1896–A1918, 2016.

[1] CNRS, École Polytechnique, France (nicole.spillane@cmap.polytechnique.fr)